Matching Students and Professors in Higher Education*

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Abstract

In higher education, various assignment rules exist for pairing students with course professors, ranging from simple random assignment approaches to mechanisms that grant students considerable choice. How do these contribute to learning and the efficient use of instruction inputs? This paper develops an econometric framework to estimate student-professor match effects and uses the framework to evaluate how students' choice of instructors contributes to learning. I extend the literature on teacher value-added by showing how to use sequences of subject-related courses to semi-parametrically identify instructor-specific learning production functions when instructors differ in grading policies and teaching abilities. The framework accommodates endogenous course selection, course dropout, and discrete scoring, all ubiquitous in higher education. Using post-secondary academic records from a university in the Dominican Republic, I estimate the model and document the existence of substantial student-professor match effects. However, when allowed to choose, students do not always select the instructor from whom they will learn the most; they place as much weight on expected grades. Relative to the current assignment rule, assigning students to the predicted learning-optimal instructor on average leads to a 4.87 percent increase in a student's academic achievement, a 5.54 percentage points reduction in the dropout rate, and a decrease from 1.53 to 1.17 instances in the number of enrollments needed for course completion.

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1 Introduction

Higher-education institutions employ various assignment rules for pairing students with course professors, ranging from random approaches to course-enrollment mechanisms offering students considerable choice. Despite aligning with some university objectives, the impact of these assignment rules on learning outcomes remains unclear. This is especially true for mechanisms relying on student choice, where results hinge on students' preferences for instructors' characteristics. For example, students may choose course professors based on expected learning, anticipated scores when grades serve a signaling role, or other factors. Each of these can result in distinct student-professor assignments and, in the presence of match effects, in different outcomes such as students' learning achievements, dropout rates, and the number of retakes required for completion of a course[1].

This paper develops an econometric framework to measure match effects in higher education learning technologies and applies it to analyzing policies that enhance learning outcomes through student-professor reassignments. I overcame two primary challenges faced by the education matching effects literature. First, as professors in higher education differ in their grading policies, it is not possible to infer instruction quality from the observed within-professor variation of scores, as is common in the elementary and secondary education literature. Second, since in many instances students can select into courses, understanding the learning outcomes of choice-based assignment rules requires modeling how students' preferences for instructors interact with the assignment rules to generate a matching. Using academic records from the *Instituto Tecnológico de Santo Domingo* (INTEC), a university in the Dominican Republic, I estimate professor-specific learning production functions for calculus instructors and quantify students' preferences for the learning and score outcomes attained under each professor. Although the estimates reveal significant match effects, students do not consistently choose their learning-optimal instructor when allowed to choose; they place as much weight on expected scores. Counterfactual simulations show that reassigning students to professors can lead to significant gains in students' learning outcomes.

Various institutional features of INTEC guide my approach. For instance, students commit to a major upon enrollment, which requires them to follow a rigid collection of subject-related sequences of courses. This means students decide on the section within a course rather than choosing among various courses, simplifying the modeling of an otherwise complex course selection problem. Furthermore, concentrating on INTEC's Calculus sequence holds empirical advantages. First, as a predominantly STEM-focused institution, nearly all students enroll in courses of the sequence. Second, the standardized

[1]Reducing dropout and retake rates are deemed as essential goals in education policy debates, for reasons that go beyond their relationship to underlying learning. For example, high dropout rates delay a student's entry into the labor market. They have also been associated with decreases in the likelihood of degree completion.

nature of the syllabus suggests variations in teaching quality are primarily vertical rather than horizontal. Third, the standalone nature of Calculus courses, mostly isolated from other math-intensive courses, mitigates concerns about interference from parallel learning. Lastly, students commence enrolling in the Calculus sequence in their initial term, ensuring no learning occurs between my measurement of pre-enrollment student characteristics and the sequence's initial enrollment. On the demand side, students' characteristics predict their demand decisions when allowed to choose. For example, Calculus 2 instructors differ in their distribution of students' initial ability, as measured by a student's score in the math component of the entrance exam score, suggesting that (when choice is allowed) students select into sections based on functions of their ability.

Central to the arguments in the paper is the significant variation in the score returns to a student's ability across instructors, consistent with matching effects in the learning technology. By itself, this observation cannot be taken as hard evidence for learning match effects, as variation in, for example, Calculus 1 score returns to ability could result from differences in instructors' grading policies rather than differences in their teaching quality. However, similar conclusions emerge when examining Calculus 2 score returns to ability for students who differ in their Calculus 1 instructor match but share a common Calculus 2 professor. As Calculus 2 scores are affected by Calculus 1's learning but not by its grading policies, at least part of the variation can be attributed to differences in Calculus 1 instructors' teaching quality[2].

To explain these observations, I construct a structural model describing students' learning outcomes along a sequence of subject-related courses in a post-secondary institution. The framework comprises two primary blocks. First, Section 5.1 introduces a model for the learning production function, describing learning outcomes and the scores they induce, taking student-professor matches as given. A student's ability is understood as endogenously evolving based on a student's learning along the sequence. The analysis extends the teacher value-added literature by modeling instructor-specific learning production functions in contexts where professors differ not just in teaching quality but also in grading policies (e.g., for early contributions, consider Hanushek (1971), Rockoff (2004), and Rivkin et al. (2005)). Additionally, it accommodates the possibility of students dropping sections of a course and accounts for the discrete nature of the reported scores, all standard factors in post-secondary environments.

While this model for the learning technology suffices for constructing counterfactual reassignment simulations, comprehending the reasons behind potential inefficiencies in the observed matches necessitates modeling the assignment rules guiding how students and professors are paired. The second component of the model, outlined in Section 5.2, delineates the assignment rules employed by INTEC. Initially, first-term students are

[2] Key to this observation is the fact that Calculus 1 and Calculus 2 are conceptually related. In other words, learning Calculus 1's material predicts a student's performance in Calculus 2

randomly matched to course sections. All remaining students must select sections of a course by engaging in a first-come-first-served course-enrollment mechanism through a platform accessible each academic term, offering access to available sections within a course. Sections are subject to capacity constraints, restricting students from selecting sections with available slots upon platform entry. I model students' decisions based on their preferences for section attributes, including the associated professor's teaching ability and grading policies. The framework extends models used to study the effects of grading policies on students' decisions by considering the decision over sections/instructors within a course, as opposed to students' decisions over major programs, courses, or study effort (i.e., examples are Ahn et al. (2019) and Babcock (2010)). Additionally, it explicitly separates learning from scores, allowing me to structurally study how grading policies and students' choices directly affect learning as opposed to other related quantities.

Section 6 presents the empirical model, outlining the key distributional assumptions guiding the estimation exercise. I parameterize learning production functions and specify learning inputs, primarily focusing on a student's ability level. By allowing variation in production function intercepts, ability slopes, and ability exponents across instructors, the model accommodates a rich class of learning technologies encompassing multiple match effect patterns[3]. On the demand side, students are assumed to select into sections based on their unobserved learning and scoring expected outcomes, with their preferences being represented by utility functions parameterized in terms of the marginal utility of each of these concerns. Given these preferences and the equilibrium disutility of participating in the course-enrollment mechanism, students decide when to enter the platform and which instructor to choose among those available upon entry. Utility section-term fixed effects capture additional factors influencing demand decisions, including section schedule and instructor attributes unrelated to the learning/scoring outcomes.

Methodologically, the main conceptual challenge is disentangling instructors' teaching abilities and grading policies from the observed distribution of scores. Proposition 6, the main conceptual result of the paper, shows how to achieve this by using sequences of subject-related courses, standard in post-secondary curricula. To illustrate, consider comparing the teaching quality of two Calculus 1 professors. Under uniform grading policies, a straightforward thought experiment shows a way to proceed: (i) pair two students with identical histories to each instructor, and (ii) assess the instructors based on their students' Calculus 1 scores. Indeed, this approach is common in the literature focusing on elementary and secondary education settings where standardized tests are available (e.g., see, for example, Ahn et al. (2019), which considers matching effects in settings other than higher education). However, it is invalid under heterogeneity in grading policies as variations in scores may stem from differences in teaching abilities or

[3]Under my parameterization, two instructor parameters interact with a student's ability in the production functions. This leads to efficient assignments taking forms that go beyond positive/negative sorting.

grading policies. Subject-related course sequences offer a solution. Intuitively, a student's performance in a subsequent course is influenced by the learning of the current course, not the grading policies of present instructors. Regarding the experiment, I can assign both students to a common Calculus 2 instructor and use the difference in the resulting Calculus 2 scores to infer teaching ability differences among the Calculus 1 professors.

In terms of the demand model, the primary challenge arises from the absence of observed entry times for students participating in INTEC's course-enrollment mechanism. Because a student's entry time determines its choice set at equilibrium, I cannot observe the set of available sections from which a student makes a demand decision. The classical identification problems resulting when preferences and choice sets are both unobserved follow. In Proposition 4, I discuss how reinterpreting the student's section demand model from an ex-ante perspective can address this problem. Intuitively, I can think of the decisions faced by a student seeking to enroll in a section of a course in terms of (i) the student chooses a section among all the sections available (the full choice set), (ii) the student chooses to enter the platform in a utility-maximizing manner conditional on ensuring availability of a slot in the section previously chosen. This interpretation differs from the ex-post perspective, where the student enters the platform first and then selects an instructor from the following restricted choice set. Although both versions of the demand model are behaviorally equivalent, the former corresponds to a straightforward discrete choice random utility model, where all students face the full choice set. Although not all primitives of the demand model can be identified under the reinterpreted model, I show that students' marginal utility for learning and scores can be inferred.

Employing the estimates, I explore how two classes of assignment rules employed by INTEC can generate learning-efficient assignments: (i) random assignment rules and (ii) first-come-first-served assignment rules. Using the second mechanism, I propose a novel channel through which heterogeneity in grading policies can affect learning outcomes for assignment rules relying on choice. In essence, given the signaling value of course scores in post-secondary education, a student's participation in a choice-based course-enrollment mechanism is likely to reflect a preference for attaining high scores alongside a concern for learning. If high-learning instructors do not coincide with high-scoring professors, the mechanism can produce inefficient matches from a learning standpoint.

The empirical exercise reveals substantial match effects within the learning technology with important implications for students' learning and scoring outcomes. For instance, students might encounter score differences equivalent to more than a full letter grade between the best and worst potential matches with professors. Additionally, the estimates show non-trivial variation in instructors' grading policies, particularly in the marginal score return related to learning. Importantly, high-learning instructors do not always coincide with high-scoring professors. On the demand side, estimates show that students have strong preferences for the scores they anticipate from an instructor match, equal in magnitude to their learning concerns and large relative to preferences over other section-instructor attributes.

For each observed course-enrollment instance, I compute the predicted number of professors delivering better scoring outcomes than the learning-optimal professor. Among all course-enrollment instances, approximately 87% of students can find at least one score-improving professor deviation, with a significant fraction of students facing multiple opportunities (i.e., approximately 44% of students face more than five potential deviations). These result in substantial score gains, with the average score gap between the scoring-optimal instructor and the learning-optimal instructor among students actively demanding a section corresponding to 1.12 points on the GPA scale. The latter, together with estimates on students' preferences, suggests a learning vs. score tension whose resolution is implied by the fact that a substantial amount of students choosing a section under the observed current assignment rule end up matched with an instructor other than the predicted learning-optimal one (i.e., 80.75% for students who participate in the first-come-first-served mechanism), suggesting room for gains via reassignments.

Counterfactual simulations of policies seeking to reassign students to professors show the magnitude of these gains. Although all considered policies coincide in the objective to maximize a weighted sum of average learning output, they differ in the weight placed over students with different ability levels to capture distributional considerations. All policies result in substantial learning gains: an average 4.87% increase in learning outcomes, a 5.54-percentage-point drop in section withdrawal rates, and a decrease from 1.53 to 1.17 instances in the average number of enrollments needed for course completion. Importantly, although varying in magnitude, the gains are positive throughout the entire distribution of student ability, implying the possibility of improving match efficiency without sacrificing distributional goals regarding learning.

The rest of this paper is structured as follows. Section 2 positions the paper's contributions within the literature. Sections 3 and 4 introduce the empirical setting, highlighting key stylized facts crucial for constructing the empirical model. Section 5 presents the conceptual model, discussing both the learning production function and the model for section demand within a course. Section 6 introduces the empirical model and outlines the identification arguments for the primitives of interest. Section 7 details the model estimates and discusses their implications for the observed assignment rule. Section 8 examines the outcomes of counterfactual reassignment policies. Finally, Section 9 concludes and suggests potential avenues for future research.

2 Related Literature

This project contributes to an extensive body of research studying heterogeneity in instructors' teaching quality. Within this group, much work has taken place under the value-added framework. Some early contributions include Hanushek (1971), Rockoff (2004), Rivkin et al. (2005), and Hanushek (2009). These are primarily empirical projects emphasizing teacher quality measures to guide hiring, promotion, and dismissal decisions in education settings. They document substantial disparities among instructors' teaching quality and gains from policies that act on such differences. In terms of recent conceptual and methodological contributions, consider Kane & Staiger (2008), Chetty et al. (2014),Gilraine et al. (2020), and Gilraine & Pope (2021).

Common to these papers is the assumption that learning production functions additively separate student and professor inputs. This modeling decision, stemming from a focus on describing differences in the average teaching quality across instructors, is reasonable given the policy questions these papers seek to answer. However, it is inadequate for studying assignment problems since, under separability, aggregate learning outcomes are independent of how instructors and students are matched. Recent value-added examples exploring the existence of matching effects in the learning technology can be found in Aucejo et al. (2018), Ahn et al. (2020), and Graham et al. (2022). Although these projects concern matching effects in the learning technology, they differ from this paper in several important ways.

The most obvious is my focus on higher-education settings. This distinction is significant because the structural differences between higher-education settings and other learning environments rule out any simple extrapolation of the conclusions derived from estimates based on the latter. Instead, these call for different modeling and econometric approaches. Examples of these differences include the heterogeneity in the grading policies used by professors to map learning into scores, the fact that only discrete scores are reported (i.e., letter scores), and the truncation in the distribution of scores resulting from students being able to drop sections of a course. Relative to value-added papers considering matching effects, the approach proposed here allows for richer forms of complementarities beyond the multiplicative separability used in many of these contributions: the only requirement is for the professor-specific learning production functions to be injective relative to its inputs.

More critical are the methodological differences in the identification of the matching effects. In particular, I show how the within-professor variations in the score distribution, used in elementary and secondary settings to identify learning technologies, cannot be used in higher-education settings where instructors differ in grading policies. Intuitively, high scores under an instructor can result from either high-quality teaching or a lenient grading policy. I provide arguments for disentangling both based on observing students in multiple periods along a sequence of subject-related courses. The approach is broad in that it can be applied to a large class of post-secondary institutions and is structural, facilitating the evaluation of counterfactual policies.

The spirit of the identification argument is similar to that in Carrell & West (2010), which considers grades in future courses as the normalization defining learning. However, their concern is not one of confounding grading policies and learning, as their setting involves standardized tests. Instead, it is about distinguishing between instructors who "teach to the test" and those who have a lasting learning impact on students. The reduced form approach followed by this contribution allows for testing the hypothesis of pedagogical differences across instructors in a post-secondary education institution but does not lend itself to counterfactual analysis. Moreover, using the empirical arguments requires a setting with standardized testing and random assignment of students to professors, an uncommon situation in most higher-education institutions.

A second branch of the education literature related to this project has focused on understanding the consequences of differences in the grading policies across instructors on students' decisions within the university. Some recent examples are Ahn et al. (2019) and Butcher et al. (2014), focusing on students' decisions regarding major choices and how grading policies impact these decisions. Another example, Babcock (2010), focuses on student effort decisions within the course as an optimal response to how grading policies affect the marginal returns to studying for students who value scores as an output. Like these papers, I adopt the perspective of grading policies potentially impacting student decisions. However, the nature of the decisions considered here is very different. Specifically, I propose a new channel through which heterogeneity of grading policies can affect learning indirectly by modifying students' course/section demand decisions and, therefore, the resulting assignment. This must be contrasted with the emphasis on describing distortions resulting from students' choices of major, courses, and effort levels followed by these contributions.

Although related, these are conceptually very different. For instance, even without matching effects in the learning technology, major and course choice distortions might arise if grading policy differences exist across courses or departments. Conversely, distortions in the assignment of students to professors within a course can occur even in environments where students have little control over their major or course requirements (as is the case in my empirical setting after the initial enrollment). Furthermore, the focus on matching effects, which requires directly modeling the production technology, places this project at the intersection between the literature studying the impact of heterogeneity in instructors' grading policies and the literature concerned with quantifying real learning differences across instructors mentioned above. Recent examples that also directly model the learning technology are Gershenson et al. (2022) and Figlio & Lucas (2004), documenting real learning consequences of grading policies professors adopt. However, these papers focus again on elementary and secondary settings under technologies that don't factor in student-professor complementarities.

Finally, this project can be related to the literature studying assignment mechanisms, particularly those concerned with course allocation problems. Although most of the papers here are theoretical, I share the concern of considering course-enrollment mechanisms that the university can directly choose to achieve different objectives. Some examples directly addressing the assignment problem in educational settings are Diebold et al. (2014), Krishna & Ünver (2008), and Sönmez & Ünver (2010).

One main difference with these papers is that the focus is almost exclusively on comparing allocation mechanisms under preference-based criteria. For example, the idea is very often to set up rules that lead to a student/course assignment satisfying notions of efficiency, fairness, and stability regarding students' preferences. This approach, while reasonable, is not the only one. One can easily entertain ranking the assignment mechanisms regarding the learning outputs they induce. Indeed, learning considerations are very often the stated goals guiding universities' decisions. The approach pursued here also differs in that instead of considering the construction of mechanisms satisfying specific properties, I compare existing mechanisms by using the estimates for the primitives in the model. An exception to the latter is Budish & Cantillon (2012), which considers, under a preference-based approach, the comparison of course-enrollment mechanisms used in a concrete empirical setting.

3 Institutional Background and Data

Section 3.1 outlines INTEC's organizational rules governing students' outcomes and decisions along the Calculus sequence considered. Section 3.2 provides formal definitions for the variables considered in the empirical exercise, particularly those linked to a student's learning outcomes and course enrollment choices.

3.1 Institutional Background

INTEC is a medium-sized university for the Dominican Republic standard, accommodating an average student body of approximately 5,697 students per academic term. The organization of learning aligns with standard higher-education norms: (i) completing a major program requires specific courses, (ii) related courses have prerequisites or corequisites, and (iii) grading employs a continuous scoring system, reflected in letter grades and a 4.0-based GPA on transcripts. However, three distinct aspects distinguish it from other post-secondary settings. While not crucial for the subsequent arguments, these aspects influence my modeling decisions and merit discussion.

First, at INTEC, students must select a major upon enrollment, committing to a predetermined sequence of courses known as the major's pensum for program completion. Notably, the pensum dictates the specific courses and their order for enrollment, offering students flexibility only within the specified course sections. This contrasts sharply with the North American higher-education model, where students have considerable flexibility in the courses they enroll in to fulfill the requirements of a major program.

Second, there are indications pointing to an absolute scoring system at INTEC rather than a relative one. For instance, professors don't face university-mandated target score distributions, and there's a lack of strong incentives for implicitly coordinating around one. This reflects an institutional culture where professors hold considerable autonomy in assessing student learning and where the score distribution within a section is perceived to convey meaning beyond just the rank order of students. Later sections in this paper present evidence ruling out some other forms of relative grading policies. For example, score distributions for specific professors shift over time, indicating a lack of professor-specific target distribution. Furthermore, variations exist in section statistics—like mean and variance—under a particular instructor. This suggests that professors don't aim to target simpler statistics instead of the entire distribution of letter scores.

Lastly, INTEC employs an assignment rule that combines random allocations and choice-based selection. Initially, first-term students are randomly assigned to course sections. Subsequently, all other students utilize a first-come-first-served mechanism to choose sections within a course. Operationally, an online platform opens each term at a specified time, displaying all available sections. Students access the platform to select sections of the courses they seek to enroll in. Importantly, sections are subject to capacity constraints, so students may be unable to enroll in a specific section if its capacity is met by the time they access the platform[4].

My analysis focuses on INTEC's first two courses along the Calculus sequence: Calculus 1 and Calculus 2[5]. Several empirical advantages result from this choice. First, most students enroll in STEM programs, and consequently, many students are observed enrolling in these courses. Second, almost no differences exist in the course syllabus across instructors. This is ideal, given my interest in measuring vertical teaching differences

[4]INTEC guarantees that the overall number of slots in a course exceeds the total demand. Thus, while students might risk not enrolling in a specific section, they don't risk finding any available slot for a given course.

[5]Calculus 1 covers pre-calculus topics, emphasizing algebra and analytical geometry. Calculus 2 is a standard first course in differential Calculus.

across instructors (i.e., teaching quality) instead of horizontal differences. Thirdly, as Calculus serves as a foundational course for many other subjects, these courses are typically taken in isolation from other math-intensive subjects, minimizing concerns about interference bias from students learning Calculus material from instructors in other math-related courses. Finally, students begin enrolling in Calculus courses in their first academic term, implying no learning between my measurement of pre-enrollment learning-related variables and the beginning of the course enrollment along the sequence. This allows for the use of pre-enrollment variables, in particular a student's initial ability, to accurately describe his understanding of the Calculus material before enrolling in the first course in the sequence.

3.2 Variable Definitions

INTEC's academic records can be categorized in terms of three groups. First I observe student-level variables related to learning. The second set contains course outcomes for each course-enrollment instance observed in the data. For each instance I observe whether the student chooses to complete the section or if instead the student chooses to dropout. The third component describes the conditions under which a section of a course is offered including both information specific to the section and information corresponding to the instructor leading the section. Below is a description of the main variables and definitions used in subsequent sections.

Academic term.— INTEC's academic calendar is divided into four terms within each year. Each term is structured in terms of ten weeks of lecturing and two weeks for evaluations. The observed sample spans academic terms between 2007 to 2022 of which I use academic terms between 2011 and 2022 for the estimation exercises.

Student initial ability.— A student's initial ability is measured as its score on INTEC's entrance math exam, the "Prueba de Aptitud Académica" (PAA). This is a college board designed test used to asses a student's understanding of the prerequisites for first year undergraduate courses. Scores, initially ranging from 0 to 800, are scaled down to a 0 to 4 scale (i.e., so that they agree with the range used by INTEC to report course GPAs). The average score in the exam under the normalization is 2.82 with a standard deviation of 0.54.

Student covariates.— Student pre-enrollment variables, recognized in the education literature as potential predictors for learning, are observed in the data. Both a student's *sex* and chosen *major* are observed. Major programs are classified into four subject areas, closely mirroring INTEC's own departmental divisions: (i) STEM, (ii) business and social sciences, (iii) health sciences, and (iv) others.

Instructor covariates.— I create dummy variables to categorize instructors in a given academic term as either being below or above the median regarding various learning-related metrics. Three dimensions are considered. The *teaching load* variable captures the total number of sections taught by a professor in the current academic term, irrespective of the course. *General tenure* captures the overall tenure of an instructor at the university in terms of the total number of academic terms the instructor has been active prior to the term being considered. Lastly, the *course-specific tenure* variable distinguishes between an instructor's overall teaching experience and its experience teaching a specific course in the Calculus sequence. The measure mirrors *general tenure* but only accounts for the previous academic terms in which the instructor has taught the specific course under consideration.

Course scores.— A *course score* is reported for each course-enrollment instance in the data. For students who choose not to drop the section of the course a letter score (i.e., A, B+, B, C+, C, D, and F) associated to a numerical score (i.e., 4.0, 3.5, 3.0, 2.5, 2.0, 1.0, 0.0) is reported. Scores D and F are deemed insufficient for a pass score and require the student to enroll the same course again in a subsequent term. Conditional on dropping a section of a course a R letter score is reported. This is associated to no numerical score.

3.3 Descriptive Statistics

Course-enrollment level descriptive statistics.— Table 1 presents the score distribution for various subpopulations in the dataset, providing statistics that average over course-enrollment instances. On the 4.00 GPA scale, Calculus 1 averages around 2.80 points, and Calculus 2 around 2.75. Examining course-enrollment instances within subpopulations based on student and professor covariates reveals significant variation. The second panel depicts score variation conditional on a student's initial ability, the main student input in my setting. For Calculus 1, scores range from 2.41 to 3.22 GPA points, moving from the first quartile to the top quartile in initial ability. Similar patterns emerge for Calculus 2.

Each succeeding panel in the table dissects the distribution according to a specific conditioning variable. For instance, the third and fourth panels illustrate the score distribution conditioned on a student's sex and major choice variables. As noted in previous research, both variables show correlations with the observed scores. Comparable findings arise in the last panel, where averages and standard deviations of scores are computed based on the teaching load, course-specific tenure, and general tenure of the instructor leading the section associated with the considered course-enrollment instance.

Section level descriptive statistics.— Table 2 supplements the previous table,

Table 1: Descriptive Statistics - Course-Enrollment Level

	Calculus 1		**Calculus 2**	
	Avg.	Std. Dev.	Avg.	Std. Dev.
All students	2.80	1.17	2.75	1.07
Ability 0% - 25%	2.41	1.21	2.45	1.09
Ability 25% - 50%	2.65	1.21	2.55	1.07
Ability 50% - 75%	2.82	1.13	2.69	1.07
Ability 75% - 100%	3.22	0.99	3.08	0.99
Female	2.94	1.10	2.87	1.01
Male	2.70	1.22	2.66	1.11
Stem	2.81	1.19	2.74	1.09
Social sciences	2.77	1.16	2.59	1.09
Health sciences	2.78	1.16	2.86	1.00
High load	2.70	1.17	2.78	1.07
Low load	2.90	1.17	2.73	1.08
High course tenure	2.96	1.10	2.85	1.04
Low course tenure	2.72	1.20	2.68	1.09
High tenure	2.92	1.11	2.85	1.05
Low tenure	2.72	1.21	2.67	1.09

Notes: Statistics in the table correspond to the course-enrollment instance level. In the final panel, high/low are defined in terms of above/below average for each of the variables being considered.

providing summary statistics at the course-section level. This perspective is crucial for understanding the variation across sections of a common course as perceived by students when making enrollment decisions.

In the first panel, both Calculus 1 and Calculus 2 exhibit an average section size of approximately 33 students. Notably, for Calculus 1, only 27% of observed sections operate at full capacity, while this percentage increases to 40% for Calculus 2. As we will see in subsequent sections, this slackness grants a planner the opportunity to address efficiency considerations via reassigning students without the need to sacrifice distributional concerns resulting from teh need to break some matches in order to meet the capacity constraints. Shifting attention to the second panel of Table 2, the focus turns to the distribution of scores at the section level. About 63% of students who enroll in both Calculus 1 and Calculus 2 successfully complete the course. The average mean section score is 2.70 GPA points for Calculus 1 and 2.66 for Calculus 2. However, it is noteworthy that a substantial proportion of students, constituting 25% in the case of Calculus 1 and

27% for Calculus 2, choose to withdraw from the initially enrolled section, leading to the absence of a final score for these students.

Table 2: Descriptive Statistics - Course/Section Level

	Calculus 1		**Calculus 2**	
	Avg.	Std. Dev.	Avg.	Std. Dev.
Section size	33.35	9.17	32.89	10.54
% at capacity	27.00	–	40.00	–
Mean score	2.70	0.69	2.66	0.56
Pass rate	0.63	0.24	0.63	0.22
Drop rate	0.25	0.21	0.27	0.19
Load	2.77	1.36	2.54	1.40
Course tenure	9.63	8.73	10.27	9.60
General tenure	11.41	9.21	12.38	10.3

Notes: Statistics in the table correspond to the course-section level.

In terms of the distribution of instructor characteristics across sections, on average an instructor responsible for a Calculus 1 section is subject to a teaching load of 2.77 courses, while for Calculus 2 the number is 2.54. Regarding course-specific and general tenure, the average Calculus section is taught by a professor with approximately 9.63 terms of experience for Calculus 1 and 10.27 terms for Calculus 2. Additionally, when considering general tenure, the corresponding figures are 11.41 and 12.38 terms.

4 Descriptive Results

This section discusses some descriptive results, highlighting the variations to be considered in the main empirical empirical exercises. Two main patterns emerge: (i) grading practices are inconsistent with various forms of relative grading policies, and (ii) instructors show differences in their scoring patterns in response to student abilities.

Heterogeneity in grading policies.— The empirical model introduced in Section 5 approximates professors' mapping of learning into scores by using absolute grading policies. Indeed, some forms of relative grading policies can be ruled out directly from the observed distribution of scores. For instance, consider examining the variation in the fraction of students obtaining a given letter score under each professor. If all instructors adhere to the same target score distribution, the proportion of students receiving each letter grade should remain consistent across professors.

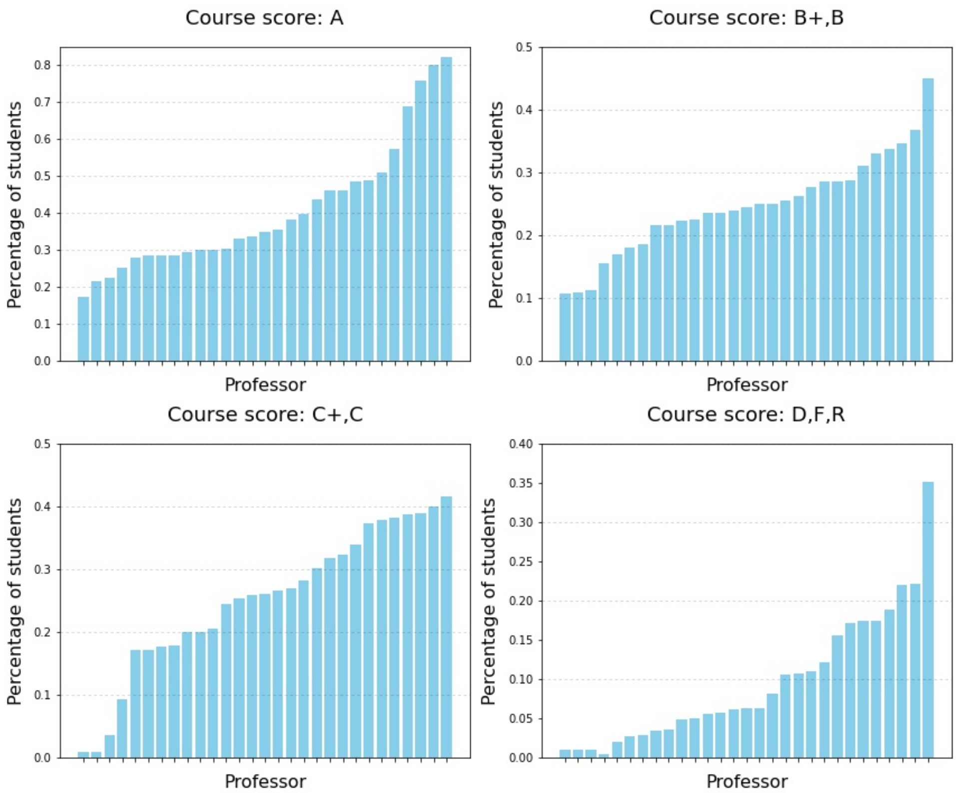


Figure 1: Fraction of students with a letter score under each Calculus 1 professor.

Figure 1 displays these fractions for Calculus 1 instructors across various letter scores. Each panel represents a different letter score, where each instructor is depicted by a bar indicating the fraction of students achieving the corresponding panel's letter score. All panels indicate significant variation in the letter score distributions across professors. Take the upper-left panel, depicting the proportion of students earning an A under each instructor. These fractions range notably, from 17% to 80%, showcasing substantial variation. Similar disparities exist across other letter scores as well.

Other forms of relative grading policies can also be shown to be inconsistent with the data. For example, consider instructor following an individual target distribution for their own sections. Figure 2 depicts, for different fixed Calculus 1 professors, the distribution of letter scores associated to multiple academic periods. In particular, consider the six instructors with the highest number of enrolled students, and the distribution for the first five academic terms in which each instructor is active. Intuitively, no variation in the distribution across academic terms would be consistent with a within-professor target distribution. Figure 2 is inconsistent with such behavior as all instructors exhibit non-trivial differences in the distribution of scores from one period to another.

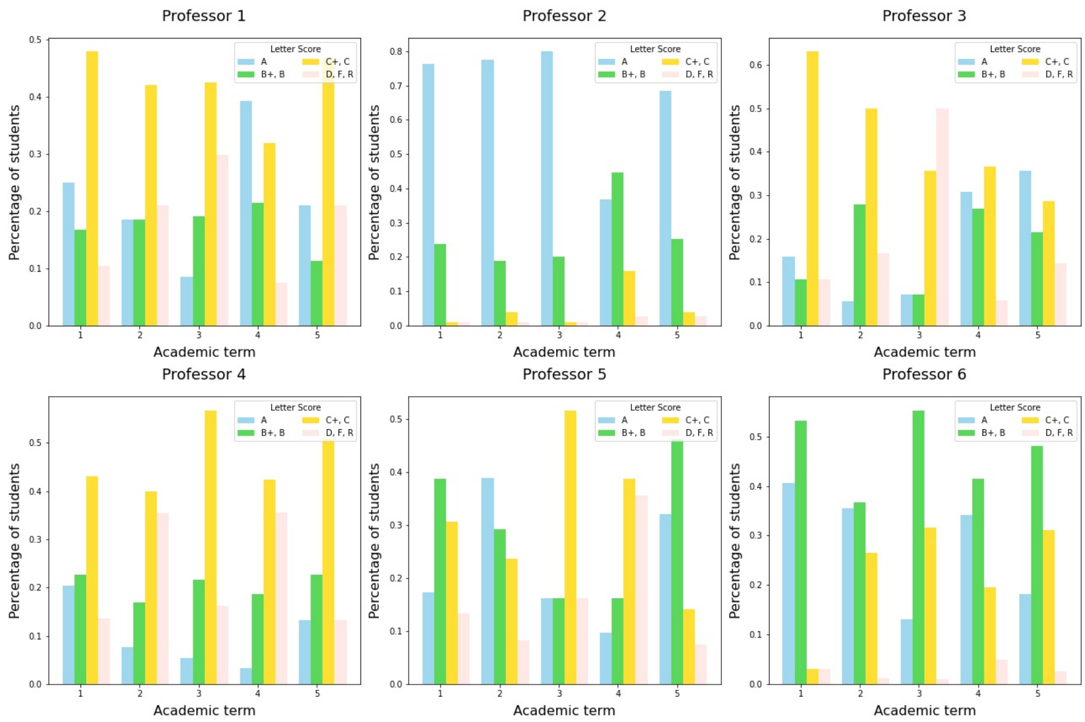


Figure 2: Within professor distribution of scores across academic terms.

Professor-specific returns to ability.— Arguments in the paper are centered on the existence of matching effects, specifically student-professor complementarities concerning a student's ability level. In such a case, variations in how increases in a student's ability impacts learning should be expected across instructors. Consider first measuring these returns using course scores as a proxy for such learning. Conceptually, I entertain an experiment involving two students with different ability levels each paired with a given instructors. Subsequently I compare how different professors lead to different score gaps between the two students. The following regression exercise captures the intuition above.

$$\text{Score}_i^1 = \gamma^0 \, + \, \gamma^1 \cdot a_i \, + \sum_{j_1 \neq 1} \gamma_{j_1}^1 \cdot (a_i \times d_{i,j_1}) \, + \, \boldsymbol{\gamma}^2 \mathbf{z}_i \, + \, \varepsilon_i.$$

Above, Score_i^1 is a dummy variable for whether student i obtains a previously specified score in Calculus 1. Let's entertain two specific outcomes for Calculus 1: "student i obtains a score of A" and "student i obtains a fail score". The model links the likelihood of a student obtaining these scores to a linear function of the students' characteristics: its ability level, denoted as a_i, and a vector of learning-related covariates, represented by $\mathbf{z}_i$. Parameter γ^1 captures the returns to ability under the excluded instructor. To accommodate the possibility of other professors differing in their returns, I consider the dummy variable $d_{i,j}$, for whether student i is matched with professor j, and write the reduced form model in terms of professor-specific ability slopes deviations from the excluded professor: γ_j^1. Figure 3 plots the distribution of the γ_j^1 coefficients.

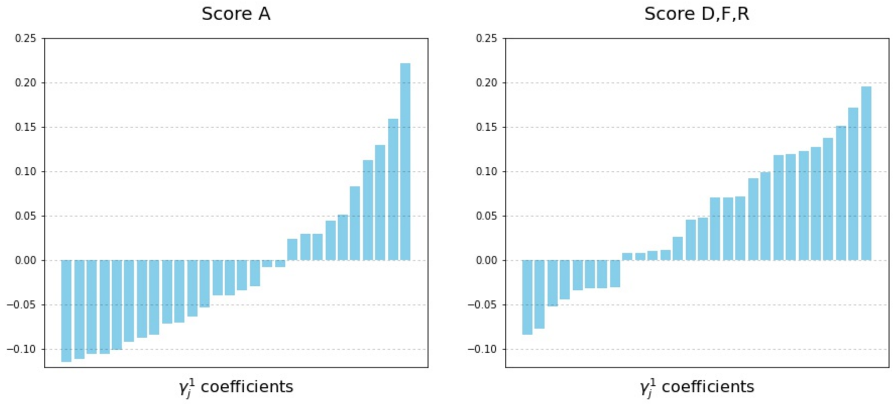


Figure 3: Distribution of γ_j^1 coefficients.

Each bar corresponds to a different coefficient associated to a given instructor. The height of a bar captures deviations the ability slope of such professor relative to the excluded instructor. Additionally, the estimates are constructed using only information on first time students as a way of mitigating potential selection concerns. Clearly nontrivial differences exist in the professor-specific slopes to ability. For example, in the first panel, when comparing the smallest coefficient and the largest coefficient the difference corresponds to approximately 0.30 so that an increase of 1.0 in a students initial ability leads to almost a half a letter jump in the likelihood of obtaining an A score under the top instructor relative to the bottom one. Consider for instance testing the null hypothesis of all the γ_j^1 coefficients being equal to zero (i.e., no differences in the returns to ability across professors). For both regressions the null is rejected (i.e., $\text{pval}_1 = \text{pval}_2 = 0.00$).

One concern is that differences in the slopes shown above could stem from differences in grading policies, as opposed to actual learning outcomes. The following thought experiment suggests an alternative exercise that deals with this issue: Imagine two students with the same entrance exam score. Enroll both of them with different Calculus 1 instructors, and after completing Calculus 1, have both students enroll under a common Calculus 2 instructor. Under the "ceteris paribus" assumption, since the only distinguishing factor in the paths these students follow is their Calculus 1 instructor, any differences in their Calculus 2 performance should reflect disparities in the learning outcomes associated with their respective Calculus 1 instructors. I can frame the experiment above in terms of the reduced form model described below. It mimics the previous regression model except for two things. First, I focus on students following a Calculus 1/Calculus 2 path coinciding in the Calculus 2 instructor. Second, instead of comparing the students in terms of their Calculus 1 scores, I compare them in terms of their Calculus 2 scores, Score_i^2.

$$\text{Score}_i^2 = \gamma^0 + \gamma^1 \cdot a_i + \sum_{j_1 \neq 1} \gamma_{j_1}^1 \cdot (a_i \times d_{i,j_1}) + \boldsymbol{\gamma}^2 \mathbf{z}_i + \varepsilon_i.$$

Figure 4 shows again the distribution of the γ_j^1 estimates for the latter regression. Differences in the ability slopes persist. In particular, the null hypothesis of homogeneous ability slopes is again rejected for both exercises (i.e., $\text{pval}_1 = \text{pval}_2 = 0.00$). The subsequent sections explore this variation as a source of identification for the professor-specific learning production functions in the structural model.

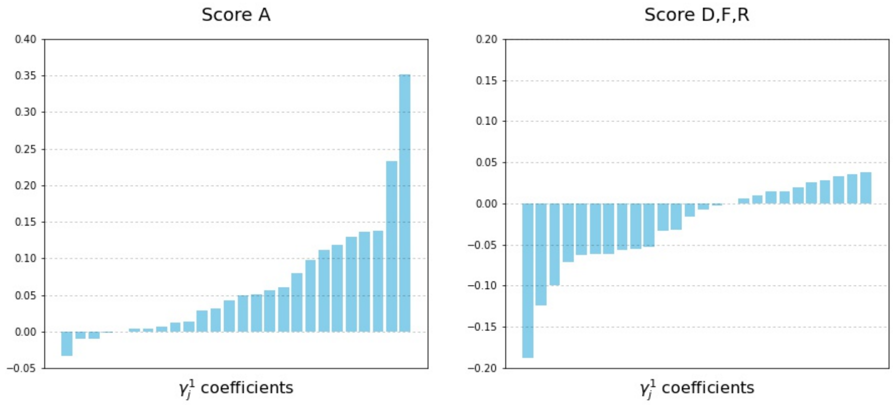


Figure 4: Distribution of γ_j^1 coefficients.

5 The Model

This section introduces a model that describes a student's academic outcomes along a sequence of compulsory subject-related courses. The model is divided into two key components. First, we outline the process through which learning takes place along the sequence, recognizing both the cumulative nature of learning, and how it depends on the interaction of both student and instructor inputs. Second, we model how students enroll in sections of a course, given the institutional constraints governing our empirical context. The aim is to construct a conceptual version of the model and to leave the formulation of an empirical version to subsequent sections.

5.1 The Learning Production Function

Consider a university that, in each academic term, denoted by $t \in \mathcal{T}$, faces the task of assigning students to instructors leading specific sections of courses in which students seek enrollment. We index an arbitrary student by $i \in \mathcal{I} \equiv \{1, ..., N\}$, and an instructor by $j \in \mathcal{J} \equiv \{1, ..., J\}$. The focus is placed on a sequence of courses centered around a common subject. These courses, which we denote by $\kappa \in \mathcal{K} \equiv \{1, 2, \ldots, K\}$, might correspond for instance to Calculus 1, Calculus 2, Calculus 3, and so forth. Students are required to enroll and successfully complete all of the courses in the sequence in the order specified by the indexes in $\mathcal{K}$. For instance, a section of course $\kappa > 1$ can be enrolled only after obtaining a pass score for course $\kappa - 1$ (e.g., achieving a pass score in Calculus 1 is a prerequisite for enrolling in Calculus 2). This sequential arrangement reflects the curriculum constraints set by the university.

Let $t_i \in \mathcal{T}$ represent the academic period in which student i enrolls in the university. Upon enrollment, i draws an ability, denoted as $a_{i,0} \in \mathbb{R}+$, which we occasionally refer to as i's initial ability type. The value $a_{i,0}$ can be interpreted as i's understanding of the prerequisite material required for the courses in $\mathcal{K}$. For instance, it may correspond to the student's score in the math component of a college entrance examination designed to assess a student's understanding of high-school pre-calculus. As the student progresses along the sequence of courses, its ability is updated in a way that reflects i's acquired knowledge of the sequence curriculum. We denote a student's type at the end of period $t > 0$ as $a_{i,t} \in \mathbb{R}_+$.

In any given academic term t, multiple sections of a course κ may be offered, with each section being guided by a single instructor. The pool of all such instructors is denoted by $\mathcal{J}_t^\kappa \subseteq \mathcal{J}$. Notice that this is potentially a strict subset of $\mathcal{J}$ as some instructors might not be active in certain periods for exogenous reasons. The multiplicity of instructors under a common course implies that more than one way of matching students to professors will exist in any given course/period pair. Let $\kappa_{i,t}$ stand for the course in the sequence student i seeks to enroll in period t, and $j_{i,t}$ for the instructor student i is paired with. While subsequent subsections describe the process by which these assignments take place, our interest here is in describing the academic outcomes conditional on the student's match.

With this goal in mind, let's consider a student, denoted as i, who, in period t, is paired with instructor $j_{i,t} = j$ for course $\kappa_{i,t} = \kappa$. Two potential academic outcomes might arise. First, i might decide to drop the section of the course, in which case an R (i.e., the notation represents 'retire') score is recorded. Such a situation is considered an unsuccessful attempt at completing the course, requiring i to enroll in the same course again in a subsequent term. The dummy variable $R_{i,t}^\kappa$ records i's decision not to drop the section of course κ (i.e., $R_{i,t}^\kappa = 0$ corresponds to dropping the section). Alternatively, the student might choose to complete the course, resulting in a discrete course score (analogous to the A, B+, B, and so forth system common in higher education institutions). Student i's discrete score upon completing the course is recorded by the discrete random variable $S_{i,t}^\kappa$. The setting described above is formally captured by the following collection of equations,

$$
\begin{array}{llll}
[\,0\,] & a_{i,t_i} & \sim F_a(\cdot), \quad j_{i,t}=j, \quad \kappa_{i,t}=\kappa, \\
[\,1\,] & a_{i,t} & = f_j(a_{i,t-1}, \mathbf{x}_{i,j,t}), \\
[\,2\,] & s_{i,t} & = \beta_j \cdot a_{i,t} + c_j, \\
[\,3\,] & R^{\kappa}_{i,t} & = \mathbf{1}\big\{s_{i,t} + \tilde{\varepsilon}^{\kappa}_{i,j,t} \geq s_{l^*}\big\}, \\
[\,4\,] & S^{\kappa}_{i,t} & = \sum_l s_l \cdot \mathbf{1}\big\{s_{l+1} > s_{i,t} + \tilde{\eta}^{\kappa}_{i,j,t} \geq s_l\big\}.
\end{array}
$$

To fix ideas, suppose a student i enters academic term t with an ability type given by $a_{i,t-1}$. Equation [1] describes the learning output of such a student after being paired with instructor j for course κ. This quantity, unobserved by the researcher, is denoted by $f_j(a_{i,t-1}, \mathbf{x}_{i,j,t})$. Notice that besides the student's ability, learning outputs are affected by a vector $\mathbf{x}_{i,j,t}$ capturing learning-related covariates. That these learning production functions are indexed by j implies the possibility of different learning outputs across instructors even conditional on the values of $a_{i,t-1}$ and $\mathbf{x}_{i,j,t}$[6]. In turn, equation [2] describes the score outcome the student obtains, $s_{i,t}$. The latter differs from learning in that it is expressed in terms of the grading policy of i's professor, $(\beta j, c_j)$.

Equations [4] describe how i's learning output maps into a course discrete score. Intuitively, we can think of $s_{i,t}$ as the student's expected continuous score obtained in period t. To make it clear that such a score depends on the student's ability and the underlying covariates, we will occasionally use the notation $s_{i,t} \equiv s_j(a_{i,t-1}, \mathbf{x}_{i,j,t})$. Notice that this quantity differs from $S^{\kappa}_{i,t}$, the discrete score obtained by the student. While the former represents the instructor's granular assessment of the student's performance (e.g., the 100 points based raw score i obtains in j's course) the latter is a discrete variable indicating the region of the score support where $s_j(a_{i,t}, \mathbf{x}_{i,j,t})$ falls. Institutional rules fix thresholds $s_1 > s_2 > ... > s_L$ which determine the map between a student's continuous underlying score and its final discrete score. As an example, student i obtains a score of s_l if $s_j(a_{i,t}, \mathbf{x}_{i,j,t})$ (plus a random perturbation) exceeds the threshold s_l but falls short of the threshold for score s_{l+1}. The error terms $\tilde{\eta}^{\kappa}_{i,j,t}$ and $\tilde{\varepsilon}^{\kappa}_{i,j,t}$ perturb the relationship between a student's continuous and discrete course scores.

We highlight that $s_{i,t}$ depends not only on the learning generated by the match but also on the instructor's grading policy (β_j, c_j). We can interpret these as encapsulating the leniency or stringency with which a student's learning is evaluated in the course. Figure 5 depicts this by plotting the map $a_{i,t} \to \beta_j \cdot a_{i,t} + c_j$ for two different instructors who differ only in their grading policies (i.e., but whose learning production functions coincide: $f_j = f_{j'}$). For instance, the blue curve depicts an instructor who, while more lenient in terms of the marginal return to learning (e.g., a higher β_j), is more stringent in

[6] This can be interpreted in terms of allowing for pedagogical differences across instructors which are not captured by any of the measured inputs of the learning production functions.

terms of the level of the scoring equation (e.g., a smaller c_j). These differences map two students, with the same underlying learning output, to different scores under each of the professors. For example, while under the red curve students with low ability levels end up above the s_l threshold, the same is not true under the scoring equation corresponding to the blue curve.

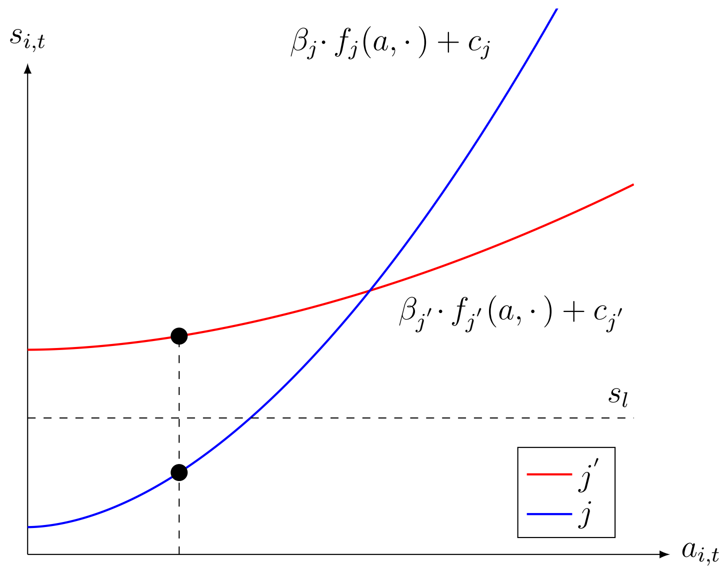


Figure 5: Grading policy differences

As explained before, student i can choose to drop instructor j's section of course κ which is explained by equation [3]. Intuitively, i chooses to drop the section whenever its underlying continuous score, $s_j(a_{i,t}, \mathbf{x}_{i,j,t})$, places him below a certain threshold s_{l^*}. As an example, one can think of a student choosing to drop the course whenever it expects to end up with a fail score. Notice that the error term $\tilde{\varepsilon}^{\kappa}_{i,j,t}$ allows for heterogeneity in the course dropping threshold. It can also capture uncertainty resulting from course dropping decisions depending on noisy signals of the true underlying score (i.e., the student's perception of its position in the grading policy after the midterm, but without the final exam signal). An error term $\tilde{\varepsilon}^{\kappa}_{i,j,t}$ with a large variance could for instance correspond to a situation in which students face a lot of uncertainty before the dropout deadline. Correlations between $\tilde{\varepsilon}^{\kappa}_{i,j,t}$ and $\tilde{\eta}^{\kappa}_{i,j,t}$ describe unobserved relationships between the scoring and course dropping outcomes[7].

We conclude our description by being precise about the units under which learning is being measured. In a setting described by standardized testing, all students are tested under a common grading policy so that a natural choice is to measure $f_j(a, \mathbf{x})$ in the

[7]Following the midterm/final analogy, we can interpret correlations between the error terms as capturing the fact that the student's position after the midterm is a noisy signal of what its final continuous score will be. In other words, a student who outperforms its own type after the midterm is also likely to do so for the final exam.

units of the common test. This is not the situation in our empirical context as in higher education institutions students are evaluated according to the grading policy of their matched instructor. We instead propose defining learning output in terms of the grading policy of a reference professor $\hat{j}^{\kappa}$ for each course κ[8]. It then follows that for any course κ instructor j, we interpret $f_j(a, \mathbf{x})$ as the learning output of a student who is instructed by professor j but graded according to instructor $\hat{j}^{\kappa}$'s grading policy. We can then interpret (β_j and c_j) as deviations of instructor j's grading policy from that of the reference professor. Under this intuition, c_j corresponds to the baseline score granted by the instructor and β_j captures the marginal reward to learning under instructor j (i.e., in both cases relative to the reference professor). It is immediate that under the proposed normalization, $\beta_{\hat{j}^{\kappa}} = 1$ and $c_{\hat{j}^{\kappa}} = 0$.

5.2 The Demand for Sections Within a Course

The preceding subsection provides a model for how learning and related academic outcomes are determined given a student-instructor match. Now, we describe how these matches emerge in our empirical setting. Two rules govern the assignment of students to professors at Intec. First, all first-period students (i.e., students in their initial academic period) are randomly assigned to a section of course $\kappa = 1$. Second, all other course enrollment instances, require the student to enroll a section of a course κ by participating in a first-come-first-serve mechanism. To be precise, every academic term t a course enrollment platform will open enabling students to enroll in a section of the course. Since multiple sections can be associated with the same instructor, we must introduce additional notation that distinguishes two sections under the same instructor. In particular, consider denoting a particular section as $s \in Sec_t^{\kappa}$, where Sec_t^{κ} represents the collection of all sections of course κ active in period t. Of course, each of these sections must be under an instructor j in $\mathcal{J}_t^{\kappa}$. Whenever it is not obvious from the context, we explicitly keep track of the professor associated with section s using the notation j_s.

The course-enrollment mechanism implies that a student attempting to enroll in a section of course κ faces two sequential decisions. First, student i must choose an entry time, $\tau > 0$, to access the platform. Subsequently, i must select, from the available sections, which one to enroll in. These two problems are intertwined since sections are subject to capacity constraints (i.e., limited slots are available for each section due to institutional constraints). This implies that students may need to access the platform early to secure enrollment in highly demanded sections. Formally, we frame the decision problem of a student i seeking a section of course κ in terms of the following two-stage optimization problem,

[8]The identity of the reference professor is irrelevant and any course κ professor can serve this role.

$$\max_{\tau \geq 0} \left[\max_{s} \left\{ U_{i,s,t} \text{ s.t. } s \in \mathcal{C}_t(\tau) \right\} + \phi(\tau) \right],$$

$$\mathcal{C}_t(\tau) \equiv \{ s \in Sect_t^{\kappa} : \tau \leq \tau_{s,t}^{eq} \}.$$

The inner maximization problem corresponds to a standard discrete choice problem, where students choose the section of the course that maximizes their utility, denoted by $U_{i,s,t}$. Importantly, students can only select sections from the set $\mathcal{C}_t(\tau)$, which includes all active sections whose capacity constraint is not binding at the entry time τ. In other words, i's choice set in period t may be potentially smaller than $Sect_t^{\kappa}$, the set of all active sections in t. Since the availability of a slot in a given section depends on the demand decisions of other students, we must treat the choice set faced by i as an equilibrium object. The term $\tau_{s,t}^{eq} > 0$, assumed to be known by the students, denotes the equilibrium time at which the capacity constraint of section s becomes binding. For instance, very popular sections will be associated with small values of $\tau_{s,t}^{eq}$, while the opposite holds for unpopular sections.

In turn, the outer maximization problem pertains to the decision of when to enter the platform, while recognizing that this choice influences the set of options the student will ultimately face. The formulation above assumes that students face a cost from participating in the course enrollment mechanism, $\phi(\tau)$, and that such a cost is a function of their platform entry time decision. This cost rationalizes the fact that not all students choose to enter the platform at the same time and can be interpreted as a reluctance towards early enrollment or more generally of participating in the mechanism.

We adopt a random utility model approach for the inner maximization problem by treating $U_{i,s,t}$ as a random variable. This allows us to model preferences in terms of a systematic component, shared by all students with common characteristics, as well as an idiosyncratic component capturing unobserved heterogeneity in students' preferences. In particular, as seems reasonable from our descriptive evidence exercises, we assume student i's utility for section s under an instructor j takes the following form,

$$U_{i,s,t} = U_{s,t}(s_j(a_{i,t}, \mathbf{x}_{i,j,t}) \ , \ f_j(a_{i,t-1}, \mathbf{x}_{i,j,t})) + \nu_{i,s,t}.$$

Intuitively, our model for section preferences postulates that students derive utility not only from the score they expect to obtain under instructor j but also from the actual learning derived from the match. Different functional forms for $U_{s,t}(\cdot)$ can be used to capture various preferences for these two components within the student population. For example, at the extremes, students might have preferences that depend on only scores or learning. As suggested by the indexing of the systematic utility, students might also have preferences related to other aspects of the section being demanded, such as the course schedule or characteristics of the instructor, not directly related to the learning or scoring outcomes expected by the student. These preferences can be incorporated into the formulation above by using, for example, preference fixed effects as part of the systematic utility $U_{s,t}$ specification. The term $\nu_{i,s,t}$ is an error term reflecting the idiosyncratic component of utility.

While the above formulation clearly outlines the two steps involved in the course demand problem faced by students, it is also possible (and potentially advantageous from an empirical point of view) to express the demand problem from a different perspective. Namely, one can think of students first choosing a section s from the full set of active sections $Sect_t^{\kappa}$, and subsequently choosing a platform entry time that maximizes their utility conditional on securing a slot at the section choice. We can derive this alternative formulation by manipulating the expression for the demand model as in the following,

$$\max_{\tau}\left[\max_{s}\left\{U_{i,s,t}\ \text{ s.t. }\ s\in\mathcal{C}_t(\tau)\right\}+\phi(\tau)\right]=\max_{s}\left[U_{i,s,t}+\max_{\tau}\left\{\phi(\tau);\ \text{ s.t. }\ s\in\mathcal{C}_t(\tau)\right\}\right].$$

One can think of this formulation as the decision of student i from an ex-ante perspective. Before the platform opens the student face no constraints in its choice set, as it can always choose to enter the platform sufficiently early (i.e., which requires accepting the cost of such decision) in a way that ensures the availability of a slot in the section being demanded.

5.3 Discussion

In this section, we explore the identification of an empirical version of the model described above. Before delving into this, it is beneficial to compare our framework with other commonly utilized empirical models for quantifying disparities in pedagogy among instructors. This comparison will facilitate the placement of our model within the existing literature and underscore certain identification challenges arising from our divergences from these established models. To illustrate, let's consider the following model for generating learning outcomes presented below[9].

$$\begin{aligned}
&[\,0\,] \qquad a_{i,0}\sim F_a(\cdot\,),\\
&[\,1\,] \qquad s_{i,1}=f_j(a_{i,0})+\tilde{\eta}_{i,j,1}.
\end{aligned}$$

This simple model captures summarizes (in essence) a substantial body of work in assessing pedagogical disparities among instructors. For instance, assuming $f_j(\cdot\,)$ is additive separable in student and professor attributes is a common feature in many educational studies. As another example, the fact that scores are expressed in the same

[9]We use the same notation as in the model described in the preceding section.

units as the learning production function corresponds to situations in which standardized tests are available. The prevalence of such a model in the literature largely stems from its high tractability from an econometric perspective, enabling the estimation of learning returns associated with each instructor by directly analyzing within-professor score distributions. For instance, in the case above, the average score for students enrolled under instructor j given the ability type a_0 serves as a consistent estimator for the functions $f_j(a_0)$.

As is clear from our previous discussions, the model is a poor description of higher education environments, which is why we choose to deviate from it. Nevertheless, each of these deviations presents empirical challenges that render the identification approach described earlier inapplicable. Let's see this by means of some examples. To be concrete, consider a minor modification of the previous model as to account for differences in instructors' grading policies while keeping other aspects the same,

$$
\begin{aligned}
&[\,0\,] \qquad a_{i,0} \sim F_a(\cdot\,), \\
&[\,1\,] \qquad s_{i,1} = \beta_j\cdot f_j(a_{i,0}) + c_j + \tilde{\eta}_{i,j,1}.
\end{aligned}
$$

Even without considering the other elements in our framework, it is evident that the within-professor conditional average approach discussed earlier is no longer useful in identifying the learning production functions. For example, the average scores of students under instructor j conditional on the ability type $a_{i,0} = a_0$, is now consistent for a quantity that conflates both learning returns and grading policies, $\frac{1}{n}\sum_i s_{i,1} \quad \to_p \quad \beta_j\cdot f_j(a_0) + c_j$. Put simply, observing high average scores may indicate either a high learning return under professor j, a choice of a very lenient grading policy, or both. Clearly, the latter is unsatisfactory if the aim is to deduce the nature of an instructor's production function.

As a second example, consider the following alternative deviation from the model in the direction of our framework. Specifically, let's modify the model by allowing students to withdraw from previously enrolled courses/sections. Following our formulation, an example of this corresponds to the following,

$$
\begin{aligned}
&[\,0\,] \qquad a_{i,0} \sim F_a(\cdot\,), \\
&[\,1\,] \qquad s_{i,1} = f_j(a_{i,0}) + \tilde{\eta}_{i,j,1}, \\
&[\,2\,] \qquad R_{i,1} = \mathbf{1}\{s_{i,1} + \tilde{\varepsilon}_{i,j,1} \geq s_{l^*}\}.
\end{aligned}
$$

Since only the scores of students who choose not to withdraw from a course can be observed in the academic records, the approach based on the within-professor average scores of students conditional on ability must also condition on the students not choosing to withdraw from the section of the course. In this case, such an average is again consistent for a quantity that differs from the learning production function images of interest, $\frac{1}{n}\sum_i s_{i,1} \;\; \to_p \;\; f_j(a_0) + \mathbb{E}(\tilde{\varepsilon}_{i,j,1}\,|\,R_{i,1} = 1)$. The implication is that, after observing high average scores for the conditioning set, the researcher is unable to determine whether these scores reflect a high learning return or merely the fact that the average is computed for students with high $\varepsilon_{i,j,1}$ draws.

6 Empirical Model and Identification Arguments

6.1 Identifying the Learning Production Function

Consider the problem of inferring the shape of the learning production function associated to a given instructor in course $\kappa \in \mathcal{K}$. As explained before, the main challenge this poses lies in disentangling the contributions of grading policies and actual learning outputs on the observed distribution of scores. To address this, we exploit the sequential enrollment of students into courses in the sequence $\mathcal{K}$, and the fact that while learning in course κ impacts outcomes in course $\kappa + 1$, the same is not true about the grading policies used by κ instructors. The starting point is a set of assumptions regarding the distribution of the error terms in our model.

Assumption 1. *The following assumptions are assumed to hold,*

1. *Random variables $\eta^{\kappa}_{i,j,t}$ and $\varepsilon^{\kappa}_{i,j,t}$ exist such that $\tilde{\varepsilon}^{\kappa}_{i,j,t} = \sigma^{\kappa}_{\varepsilon} \cdot \varepsilon^{\kappa}_{i,j,t}$ and $\tilde{\eta}^{\kappa}_{i,j,t} = \sigma^{\kappa}_{\eta} \cdot \eta^{\kappa}_{i,j,t} + \sigma^{\kappa}_{\varepsilon} \cdot \varepsilon^{\kappa}_{i,j,t}$ for the scalars $\sigma^{\kappa}_{\varepsilon}$,$\sigma^{\kappa}_{\eta}$,*

2. *The sequences $\{\eta^{\kappa}_{i,j,t}\}_{i,j,t}$ and $\{\varepsilon^{\kappa}_{i,j,t}\}_{i,j,t}$ are mean zero and i.i.d.. Their distributions, denoted by $F_{\eta}(\cdot)$ and $F_{\varepsilon}(\cdot)$, are known by the researcher. The associated densities are denoted by $f_{\eta}(\cdot)$ and $f_{\varepsilon}(\cdot)$.*

3. *The random variable $\nu_{i,s,t}$ is independent of from $(\eta^{\kappa}_{i,j,t},\ \varepsilon^{\kappa}_{i,j,t})$.*

The first two parts of the assumption are technical and are primarily used as tools to facilitate inversion arguments in identifying the learning production functions. Essentially, we assume that the distribution of the error terms in the scoring equation and the course dropping equation can be parameterized in terms of their variances. Correlations between these error terms are integrated into the model through sums of random variables (i.e., $\sigma^{\kappa}_{\eta} \cdot \eta^{\kappa}_{i,j,t} + \sigma^{\kappa}_{\varepsilon} \cdot \varepsilon^{\kappa}_{i,j,t}$ is correlated with $\sigma^{\kappa}_{\eta} \cdot \eta^{\kappa}_{i,j,t}$). The third part of the assumption requires that unobserved heterogeneity in the preferences of students over courses/sections remains unrelated to the perturbations of the scoring and dropping equations. Although the latter involves restrictions, we aim to mitigate potential correlations by incorporating a comprehensive set of controls into our demand specification when conducting our empirical exercises.

Let's now construct an argument for the identification of the learning production function given Assumption 1. For expositional reasons, we present results for a simplified version of our model and leave a treatment of the fully fledged framework for the appendix section. In particular, we consider a version of our model in which students don't have the option of dropping a section of a course. This allows us to bypass some technical details which are not a the core of the results. Second, the focus here is on the identification of the production function primitives associated to instructors in the first course of the sequence, $\kappa = 1$. Constructing arguments for other courses will be a simple matter of adapting the notation in what follows. In addition, since our arguments will not depend on the specific time period in which a student enrolls a course/section but instead just require keeping track of whether a student is in its first or second academic term in the university, we simplify the notation by omitting the time indices. It will be clear from the context whether an argument is based on first or second term students.

With this in mind, consider the problem of identifying the learning production function of a $\kappa = 1$ instructor, j^1. Our analysis focuses on the collection of all students who in their first enrollment instance of course $\kappa = 1$ obtain a score of s_l or higher conditional on enrolling a section under j^1. Furthermore, we condition on students of an initial type $a_{i,t_i} = a_0$ and who enroll j^1's section under a vector of covariates $\mathbf{x}_1$. Our structural model offers an expression for the conditional probability described above.

$$\begin{aligned}
\mathbb{P}\big(S^1_{i,j^1} \geq s_l \mid a_0,\ \mathbf{x}_1,\ j^1\big) &= \int_\eta \mathbf{1}\big\{\beta_{j^1}\cdot f_{j^1}(a_0, \mathbf{x}_1) + c_{j^1} + \sigma^1_\eta \cdot \eta \geq s_l\big\}\ f_\eta(\eta)\, d\eta, \\
&= \int_\eta \mathbf{1}\left\{\eta \geq \frac{s_l - \beta_{j^1}\cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\eta}\right\} f_\eta(\eta)\, d\eta, \\
&= \left[1 - F_\eta\left(\frac{s_l - \beta_{j^1}\cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\eta}\right)\right].
\end{aligned}$$

In words, student i achieves a score above s_l whenever $\beta_{j^1}\cdot f_{j^1}(a_0, \mathbf{x}_1) + c_{j^1} + \sigma^1_\eta\cdot \eta^1_{i,j^1}$ (i.e., the students expected continuous score) falls weakly above the threshold s_l. The expression above just establishes a relationship between the observed mass of students satisfying the event $S^1_{i,j^1} \geq s_l$ (within the conditioning set), and a function of the primitives of the model. Under Assumption 1, we can invert the relationship to obtain the equivalent expression given below,

$$\frac{s_l - \beta_{j^1}\cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\eta} = \underbrace{F^{-1}_\eta\left[\mathbb{P}\big(S^1_{i,j^1} \geq s_l \mid a_0,\ \mathbf{x}_1,\ j^1\big)\right]}_{\equiv \theta(s_l \mid a_0, \mathbf{x}_1, j^1)}.$$

The latter lends itself to an intuitive interpretation. Consider the marginal student, whose η^1_{i,j^1} draw places him precisely at the boundary between scores s_l and s_{l-1}. Within the conditioning set, such marginal student's η draw defines the entire mass of students who ultimately receive a score above s_l within the conditioning set. For example, those with a higher η^1_{i,j^1} will achieve scores weakly above s_l, while those with smaller draws obtain a strictly smaller score. The right-hand side of the expression above identifies the marginal student's draw by finding the precise η^1 value such that the mass to its right under $F_\eta(\cdot)$ corresponds exactly to the observed share of students who obtain a score above s_l, $\mathbb{P}\big(S^1_{i,j^1} \geq s_l \mid a_0,\ \mathbf{x}_1,\ j^1\big)$ (i.e., which the econometrician can observe). Importantly, we can also identify the marginal student as that with a draw satisfying $(s_l - \beta_{j^1} \cdot f_{j^1}(a_0, \mathbf{x}_1) + c_{j^1} + \sigma^1_\eta)/\sigma^1_\eta = \eta$. The equality derived merely states that these two expressions identifying the marginal student must coincide. The notation $\theta(s_l \mid a_0, \mathbf{x}_1, j^1)$ makes it clear that any change in the conditioning arguments leads to a change in the marginal student's identity.

By itself $\theta(s_l \mid a_0, \mathbf{x}_1, j^1)$ does not offer much insight into the underlying model as it pools multiple primitives into a single expression. However, as stated in the following proposition, when considered for two different score cutoffs, s_l and $s_{l'}$, it is possible to start gaining an understanding of some primitives of interest.

Proposition 1. *The images $f_{\hat{j}^1}(a_0, \boldsymbol{x}_1)$ (i.e., $\kappa = 1$'s reference learning production function) and the variance parameter σ^1_η are point identified.*

Proof. Fixing the conditioning quantities $a_0, \mathbf{x}_1$, we can identify the marginal students associate to the letter scores s_l and $s_{l'}$,

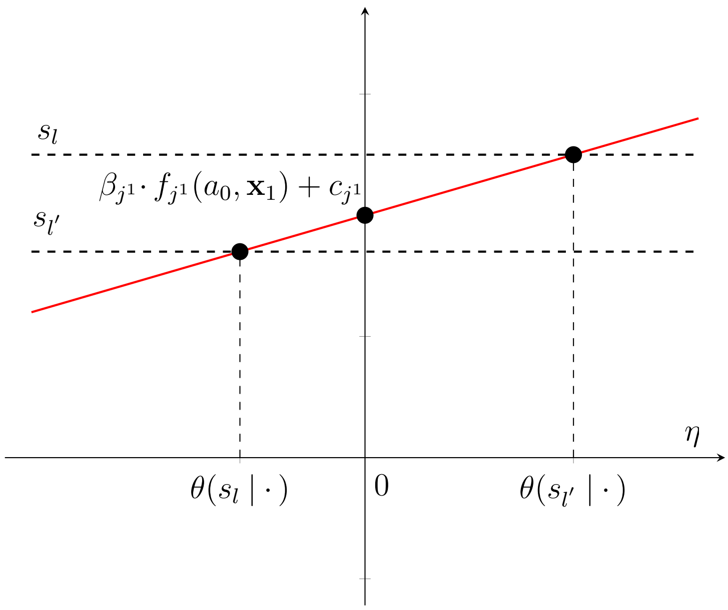
$$\theta(s_l | a_0, \mathbf{x}_1, j^1) = \frac{s_l - \beta_{j^1} f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\eta} \text{ and } \theta(s_{l'} | a_0, \mathbf{x}_1, j^1) = \frac{s_{l'} - \beta_{j^1} f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\eta}.$$

When $l \neq l'$, the latter defines a system of two equations on the unknowns σ^1_η and $\beta_{j^1} f_{j^1}(a_0, \mathbf{x}_1) + c_{j^1}$. Solving for the unique solution to the system leads to the following expression,

$$\sigma^1_\eta = \frac{s_l - s_{l'}}{\theta(s_l \mid a_0, \mathbf{x}_1, j^1) - \theta(s_{l'} \mid a_0, \mathbf{x}_1, j^1)},$$
$$\beta_{j^1} f_{j^1}(a_0) + c_{j^1} = s_l - \frac{s_l - s_{l'}}{\theta(s_l \mid a_0, \mathbf{x}_1, j^1) - \theta(s_{l'} \mid a_0, \mathbf{x}_1, j^1)} \cdot \theta(s_l \mid a_0, \mathbf{x}_1, j^1).$$

The identification of the image $f_{\hat{j}^1}(a_0, \mathbf{x}_1)$ follows from considering the second equation above for $j^1 = \hat{j}^1$ and recalling that for the reference instructor $\beta_{\hat{j}^1} = 1$ and $c_{\hat{j}^1} = 0$. ■

A graph serves as a visual representation of the argument behind the proof. Within the conditioning set, we can think of a student i's score as a linear function of its unobserved draw η^1_{i,j^1}, with an intercept of $\beta_{j^1} \cdot f_{j^1}(a_0, \mathbf{x}_1) + c_{j^1}$ and a slope of σ^1_η. Our identification of the marginal student associated to each letter score corresponds to identifying a point in the score equation. In the graph, for instance, our arguments allow us to identify the points $(\theta(s_l \mid \cdot), s_l)$ and $(\theta(s_{l'} \mid \cdot), s_{l'})$. The fact that a linear equation is pinned down by two of its points allows us to identify both the slope and the intercept of the curve. In more intuitive terms, the result states that we can always identify the learning output of an instructor j^1, in terms of its own grading policy, by directly inspecting the distribution of scores such instructor induces.



It must be emphasized that in the absence of variations in grading policies among professors, the aforementioned arguments would allow us to fully identify the learning production functions associated to each instructor. The situation resembles a scenario under standardized tests where observed scores directly reflect disparities in teaching abilities. In our context, the presence of grading policies necessitates additional efforts to separate the effects of learning returns from grading policies. With this purpose in mind let's consider the performance of students in our conditioning set in the subsequent course, $\kappa = 2$. To be precise, we are interested in the fraction of students (within our conditioning set) who after successfully completing $\kappa = 1$, enroll a section of $\kappa = 2$ under instructor j^2 and obtain a score above s_l. In addition, we focus our attention of the subpopulation of students who enroll j^2's section under a vector of covariates $\mathbf{x}_2$. As before, our model implies a concrete expression for the conditional probability of the event described above,

$$
\begin{aligned}
&\mathbb{P}\big(S^2_{i,j^2} \geq s_l, \ \mid a_0, \ \mathbf{x}_1, \ \mathbf{x}_2, \ j^1, \ j^2, \ S^1_{i,j^1} \geq s_{l^*}\big), \\
&\qquad = \left[1 - F_\eta\left(\frac{s_{l^*} - \beta_{j^1} \cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\eta}\right)\right]\left[1 - F_\eta\left(\frac{s_l - \beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0, \mathbf{x}_1), \mathbf{x}_2) - c_{j^2}}{\sigma^2_\eta}\right)\right], \\
&\qquad = \left[1 - F_\eta\Big(\theta(s_{l^*} \mid a_0, \mathbf{x}_1, j^1)\Big)\right]\left[1 - F_\eta\left(\frac{s_l - \beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0, \mathbf{x}_1), \mathbf{x}_2) - c_{j^2}}{\sigma^2_\eta}\right)\right].
\end{aligned}
$$

This expression bears a close resemblance to the one discussed earlier for the identification of $\theta(s_l \mid a_0, \mathbf{x}_1, j^1)$. The difference lies in the consideration of students who not only achieve a score of s_l or above in course $\kappa = 2$, but also those who are assigned to a specific instructor j^1 in course $\kappa = 1$ and successfully complete the course under such professor. Inverting the relationship we obtain the following identity,

$$\frac{s_l - \beta_{j_2} \cdot f_{j_2}(f_{j_1}(a_0, \mathbf{x}_1), \mathbf{x}_2) - c_{j_2}}{\sigma_\eta^2} = \underbrace{F_\eta^{-1}\left[\frac{\mathbb{P}\big(S^2_{i,j^2} \geq s_l \mid a_0,\ \mathbf{x}_1,\ \mathbf{x}_2,\ j^1,\ j^2,\ S^1_{i,j^1} \geq s_{l^*}\big)}{(1 - F_\eta(\theta(s_{l^*} \mid a_0, \mathbf{x}_1, j^1))}\right)}_{\theta(s_l \mid a_0, \mathbf{x}_1, \mathbf{x}_2, j^1, j^2)}.$$

We are now in a position that allows us to state the main result of this section. The result establishes the identification of the $\kappa = 1$ production functions under an injectivity assumption. We state and prove the result before considering an intuitive discussion of the content behind the Proposition.

Proposition 2. *The following identification results hold,*

1. *The image of the composition $\beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0, \boldsymbol{x}_1), \boldsymbol{x}_2) + c_{j^2}$ and the variance term σ_η^2 are point identified,*

3. *Suppose that $f_{j^2}(\cdot\,, \boldsymbol{x}_2)$ is injective for $\boldsymbol{x}_2$ fixed. Then the image $f_{j^1}(a_0, \boldsymbol{x}_1)$ is point identified provided the existence of $\tilde{a}_0$ such that $f_{j^2}(f_{j^1}(a_0, \boldsymbol{x}_1), \boldsymbol{x}_2) = f_{j^2}(f_{\hat{j}^1}(\tilde{a}_0, \boldsymbol{x}_1), \boldsymbol{x}_2)$.*

Proof. We start by following the same reasoning as in the previous proposition. In particular, fixing the conditioning variables $(a_0, \mathbf{x}_1, \mathbf{x}_2)$, consider the following system of equations on the unknowns σ_η^2 and $\beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0, \mathbf{x}_1), \mathbf{x}_2) + c_{j^2}$,

$$\frac{s_l - \beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0, \mathbf{x}_1), \mathbf{x}_2) - c_{j^2}}{\sigma_\eta^2} = \theta(s_l \mid a_0, \mathbf{x}_1, \mathbf{x}_2, j^1, j^2),$$
$$\frac{s_{l'} - \beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0, \mathbf{x}_1), \mathbf{x}_2) - c_{j^2}}{\sigma_\eta^2} = \theta(s_{l'} \mid a_0, \mathbf{x}_1, \mathbf{x}_2, j^1, j^2).$$

The first claim follows from noticing that when considering $l \neq l'$, the equations above define a system with a unique solution identifying both σ_η^2 and $\beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0, \mathbf{x}_1), \mathbf{x}_2) + c_{j^2}$.

Consider now the final claim in the proposition. Under the premise, we can find an ability level $\tilde{a}_0$ such that $\beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0, \mathbf{x}_1), \mathbf{x}_2) + c_{j^2} = \beta_{j^2} \cdot f_{j^2}(f_{\hat{j}^1}(\tilde{a}_0, \mathbf{x}_1), \mathbf{x}_2) + c_{j^2}$. Moreover, given the validity of the first part of the claim in the proposition, whose truth we have already asserted, we can find $\tilde{a}_0$ by directly inspecting the observed data. It follows from the injectivity of $f_{\hat{j}^2}(\cdot)$ that $f_{j^1}(a_0, \mathbf{x}_1) = f_{\hat{j}^1}(\tilde{a}_0, \mathbf{x}_1)$. However, since Proposition 1 has already established the identification of $\hat{j}_1$'s production function, the latter equality implies we can directly infer the image of j^1's production function at the argument $(a_0, \mathbf{x}_1)$. ∎

Proposition 1, our main identification argument, can be understood in terms of a simple though experiment. Suppose we observe two students with the same initial ability level a_0 but assigned to different $\kappa = 1$ instructors: student one with instructor j^1 and student two with $\tilde{j}^1$. As discussed, comparing their $\kappa = 1$ scores directly is uninformative for discerning potential gaps in their learning outputs. The disparity in the scores could be due to either instructor quality differences or differences in the grading policies used by these instructors.

A potential solution to this issue consists in comparing these two students, not in terms of their $\kappa = 1$ scores, but in terms of some other future signal related to $\kappa = 1$'s learning returns but not $\kappa = 1$'s grading policies. Carefully choosing such signals then becomes very important. For instance, one concern is that as we increase the time distance between the enrollment of $\kappa = 1$ and the measurement of the signal, the noise in the latter might increase, making it difficult to detect differences in instruction quality empirically. Moreover, one might be concerned about differences in the academic path followed by the students after $\kappa = 1$ enrollment, and prior to the measurement of the signal, which would invalidate the ideal type of ceteris paribus exercise we would like to approximate.

Given these concerns, a natural choice is to consider the scores of these students in the immediately subsequent course in the sequence. Some care is required in implementing the approach. For instance, it is reasonable to limit the comparison to students who share a common $\kappa = 2$ professor to avoid confounding effects from different $\kappa = 2$ instructors. But even then, one might be concerned about separating the contribution of this common $\kappa = 2$ professor in the observed score differences across the students. Proposition 2 states that this last point is not an issue as the contribution of the $\kappa = 2$ instructor can be filtered out from the accounting under the injectivity of its production function. Figure 6 captures this intuition graphically (while omitting from the notation $\mathbf{x}_1$ and $\mathbf{x}_2$).

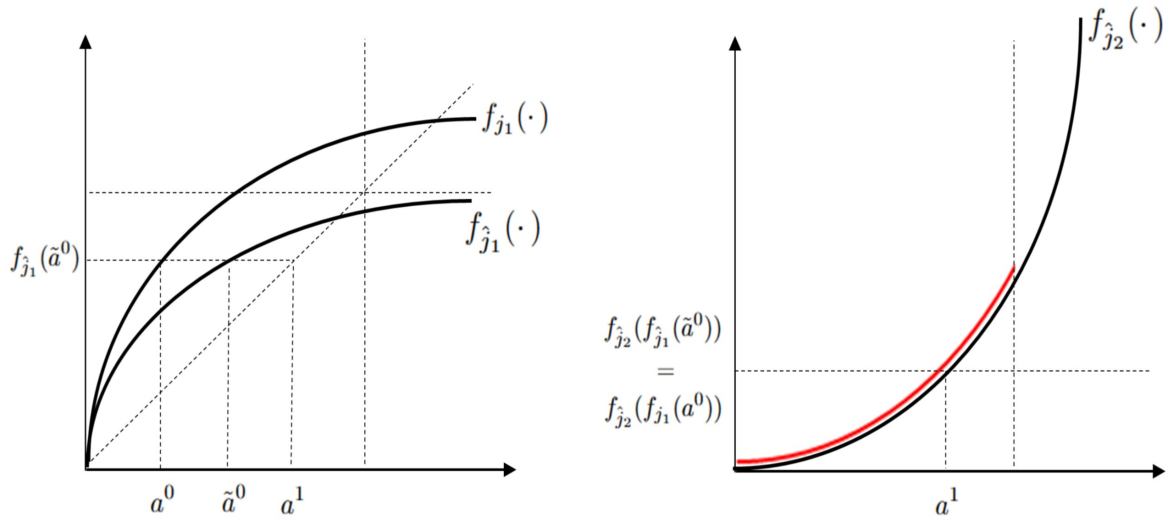


Figure 6: Identification argument for $f_j(a, \mathbf{x})$

Notice that the same arguments in Proposition 6 lend themselves to a partial identification argument in the absence of a type $\tilde{a}_0$ as required by the premise. For example, let's entertain a situation in which $f_{\hat{j}_2}(f_{j_1}(a_0, \mathbf{x}_1), \mathbf{x}_2) > f_{\hat{j}_2}(f_{\hat{j}_1}(\tilde{a}_0, \mathbf{x}_1), \mathbf{x}_2)$ for all types $\tilde{a}_0$ whose validity can be directly observed from the data. The logic of the proof above suggests there is still a lot of information we can extract from this inequality. For instance, assuming $f_{\hat{j}_2}(\cdot\,, \mathbf{x}_2)$ is monotone increasing for a fixed $\mathbf{x}_2$, we can conclude that $f_{j_1}(a_0, \mathbf{x}_1)$ must exceed the learning return induced by instructor $\hat{j}_1$ under any student $\tilde{a}_0$ in record. In other words, we can construct a lower bound for the unknown $f_{j_1}(a_0, \mathbf{x}_1)$. Similar situations can be treated in an analog way.

We conclude the subsection by highlighting that once the images $f_{j_1}(a_0, \mathbf{x}_1)$ are identified, we can identify the grading policies associated to each instructor by going back to our results on the marginal student associated to each score cutoff for $\kappa = 1$. Proposition 6.1 formally states the latter.

Proposition 3. *The grading policy of instructor j^1 (i.e., β_{j^1}, c_{j^1}) is point identified provided that $f_{j^1}(a_0, \boldsymbol{x}_1)$ is known for some $(a_0, \boldsymbol{x}_1)$.*

Proof. Consider the expressions for $\theta(s_l \mid a_0, \mathbf{x}_1, j^1)$ and $\theta(s_l \mid \tilde{a}_0, \mathbf{x}_1, j^1)$ for two different student types such that $f_{j^1}(a_0, \mathbf{x}_1) \neq f_{j^1}(\tilde{a}_0, \mathbf{x}_1)$

$$
\begin{aligned}
\theta(s_l \mid a_0, \mathbf{x}_1, j^1) &= \frac{s_l - \beta_{j^1} \cdot f_{j^1}(a_0, \mathbf{x}_1) + c_{j^1}}{\sigma^1_\eta}, \\
\theta(s_l \mid \tilde{a}_0, \mathbf{x}_1, j^1) &= \frac{s_l - \beta_{j^1} \cdot f_{j^1}(\tilde{a}_0, \mathbf{x}_1) + c_{j^1}}{\sigma^1_\eta}.
\end{aligned}
$$

It is easy to see that given the identification of the images $f_{j_1}(a_0, \mathbf{x}_1)$ and $f_{j_1}(\tilde{a}_0, \mathbf{x}_1)$, the two equations above define a system of equations on the unknowns β_{j^1} and c_{j^1}. The unique solution associated to the system identifies j^1's grading policy. ■

6.2 Identifying the Demand for Course/Sections

The preceding section introduces arguments regarding the identification of the production function model. We now turn our attention into the identification of the underlying primitives within the model for the demand of course/sections. We start the discussion the following assumptions regarding the nature of the demand model error terms.

Assumption 2. *The following assumptions are assumed to be satisfied,*

1. *$\{\nu_{i,s,t}\}_{i,s,t}$ is a collection of mean zero i.i.d. random variables whose distribution, denoted by $F_\nu(\cdot)$, is known to the researcher.*

2. *The distribution $F_\nu(\cdot)$ is continuous and of full support.*

3. *The utility function $U_{s,t}(s_j(a_{i,t}, \boldsymbol{x}_{i,j,t})\,,\, f_j(a_{i,t-1}, \boldsymbol{x}_{i,j,t}))$ takes the following functional form $U_{s,t} = \lambda_{s,t} + \alpha_0 \cdot s_j(a_{i,t}, \boldsymbol{x}_{i,j,t}) + \alpha_1 \cdot f_j(a_{i,t-1}, \boldsymbol{x}_{i,j,t}))$.*

The first part of the assumption is a standard independence assumption for the demand model error terms. Assuming a common distribution for $\nu_{i,s,t}$ across all indices, is also a standard assumption that allows us map the demand model objects to the observed instructor market shares via simple conditional choice probabilities. The second and third assumptions are technical and simply allows us to borrow some identification results from the discrete choice literature on linear random utility models.

Given these assumptions, our identification argument can be framed in terms of two main observations. First, we emphasize the implicit assumption that the cost associated with participating in the course enrollment mechanism (i.e., $\phi(\tau)$) remains uniform across all students, thereby rendering it independent of the student index i. Consequently, we can conceptualize $\max_\tau\big\{\phi(\tau);\ \text{s.t. } s \in \mathcal{C}_t(\tau)\big\}$ as a professor-period specific fixed cost incurred by a student when expressing its preference for instructor j during academic period t. This characteristic proves useful as it enables us to formulate the course demand problem as a standard discrete choice problem with utilities under alternative-period fixed effects. Denoting the sum term $\lambda_{s,t} + \max_\tau\big\{\phi(\tau);\ \text{s.t. } s \in \mathcal{C}_t(\tau)\big\}$ by $\Phi_{s,t}$, the ex-ante formulation of the demand model for a student enrolling course κ in period t can be written as follows,

$$\max_{s \in Sect_t^\kappa} \Big[\Phi_{s,t} + \alpha_0 \cdot s_j(a_{i,t}, \mathbf{x}_{i,j,t}) + \alpha_1 \cdot f_j(a_{i,t-1}, \mathbf{x}_{i,j,t}) + \nu_{i,s,t}\Big]$$

Second, we draw attention to the nature of the identification arguments put forth in the preceding section, which establish the identification of both the learning production functions and the grading policies associated to each professor. Consequently, when considering the identification of the course demand primitives, it becomes possible to regard $f_j(a, \mathbf{x})$ and $s_j(a, \mathbf{x})$ as observed quantities. These two observations significantly simplify the identification problem of the demand model, reducing it to the identification of a simple Random Utility Model (RUM). The arguments for the identification of the α_0, α_1 primitives are standard so that we can state the following result without a proof.

Proposition 4. *The parameters α_0, α_1, and $\Phi_{s,t}$ are point identified.*

7 Estimation and Results

7.1 Parameterizing the Model

As discussed above, in principle we could completely estimate the model by implementing a nonparametric estimator based on our identification arguments. While this approach offers certain desirable features, including the ability to refrain from imposing parametric assumptions on key elements such as the learning production functions, practical considerations make the idea of a more restrictive parametric approach attractive. For example, on the side of learning production, our arguments rely on the possibility of matching empirical and theoretical moments for subpopulations of students who share common academic paths. Given the relatively modest class sizes in our context, the observed student count within these subpopulations might be insufficient for empirical moments to closely resemble their theoretical counterparts. Analogous concerns may arise within the demand model.

For this reason, we consider here adopting a fully parametric approach for the estimation exercise. This adjustment not only alleviates data limitation constraints but also affords us the opportunity to specify certain parameters of interest as being common to all instructors which further reduces the data demands of the model. In what follows, we delve into the specifics of these empirical model restrictions and explain how we estimate the resulting model. The final subsection presents the estimates resulting from the approach.

Parameterizing the Learning Production Function

The specification of our empirical model commences with the parameterization of the learning production function associated with each instructor. This is guided by two principal considerations. Firstly, the functional form must exhibit enough flexibility as to accommodate a wide array of learning production shapes. Secondly, the model should be able to capture non-trivial matching effects in the learning production process. Specifically, we aim to capture interactions between instructor characteristics and our measure of student ability.

These two concerns respond to the need to allowing for flexibility at the estimation stage so that the model is capable of capturing the true shape of the learning production functions. For example, consider the additively separable specification common in empirical work, $f_j(a_0, \mathbf{x}) = \delta_j + g(a_0)$. The parameterization would be inadequate for our purposes as it would eliminate, at the modeling stage, the possibility of matching effects in the production of learning. As a second example, the multiplicatively separable parameterization $f_j(a_0, \mathbf{x}) = \delta_j \cdot a_0)$, common in theoretical settings, address the previous concern but implies simplistic positive/negative assortative matching as the only plausible learning ideal scenarios. To address these concerns, our proposal is the following: $f_j(a, \mathbf{x}) = \tilde{\delta}_j^0(\mathbf{x}) + \tilde{\delta}_j^1(\mathbf{x}) \cdot a^{\tilde{\delta}_j^2(\mathbf{x})}$. This formulation accounts for differences in the level of the learning production function, the size of the marginal returns to ability, and the nature of the returns to scale to ability. Additionally, it allows each of these coefficients to vary across instructors both in terms of observed and unobserved attributes.

To be concrete consider partitioning the covariate vector as $\mathbf{x}_{i,j,t} = (\mathbf{x}_{1,i}, \mathbf{x}_{2,j,t})$. The first component encapsulates time-invariant characteristics of students. In our estimation exercise we consider $\mathbf{x}_{1,i} = (sex_i, \{maj_{i,d}\}_{d=1}^4)$ where sex_i is a male dummy variable and $major_{i,d}$ is a dummy indicating whether student i's major choice is part of department d (i.e., we partition the set of all majors in terms of four major departments, closely following the organizational division within the university) one of four major departments in the university. Meanwhile, $\mathbf{x}_{2,j,t} = (load_{j,t}, ten_{1,j,t}, ten_{2,j,t})$. The variable $load_{j,t}$ is a binary variable specifying whether the total number of sections taught by instructor j in the academic period t exceeds a certain threshold. This allows us to account for either positive returns (learning from teaching multiple sections) or negative returns (potentially due to fatigue) associated with an instructor's teaching load in a given term. Furthermore, $ten_{1,j,t}$ and $ten_{2,j,t}$ are binary variables denoting whether or not the instructor's tenure at period t exceeds certain thresholds. The former, $ten_{1,j,t}$, reflects the number of terms of the course sequence that instructor t has taught by the beginning of period t, while $ten_{2,j,t}$ similarly measures tenure across all courses the instructor has taught in the university.

Given these considerations, we parameterize the production function coefficients in terms of the following,

$$\begin{aligned}
\tilde{\delta}_j^0(\mathbf{x}) &= \delta_j^0 + \boldsymbol{\mu}_0^{'}\mathbf{x}_{2,j,t} + \boldsymbol{\gamma}^{'}\mathbf{x}_{1,i}, \\
\tilde{\delta}_j^1(\mathbf{x}) &= \delta_j^0 + \boldsymbol{\mu}_1^{'}\mathbf{x}_{2,j,t}, \\
\tilde{\delta}_j^2(\mathbf{x}) &= \delta_j^2 + \boldsymbol{\mu}_2^{'}\mathbf{x}_{2,j,t}.
\end{aligned}$$

. Here, $\delta_j^l;\ \ l \in \{0, 1, 2\}$ are production fixed effects capturing unobserved ways in which specific instructors influence production output. In turn the terms $\mu_{\cdot}^{'}\mathbf{x}_{2,j,t}$ capture productivity differences arising from observed heterogeneity reflected in $\mathbf{x}_{2,j,t}$. Importantly, notice the t index in the latter suggesting these variables change over time, thus allowing them to be separately identified from the instructor fixed effects parameters. Finally, $\gamma^{'}\mathbf{x}_{1,i}$ allows for differences in student characteristics other than ability to affect the level of the learning production function.

To complete the description of the learning output empirical model, we must specify the distribution of the error terms in the learning production model. Given that we interpret these as perturbations of the scoring and course dropping equations, a reasonable distributional assumption is $\eta^{\kappa}_{i,j,t} \sim \mathcal{N}(0, \sigma^{\kappa}_{\eta})$ and $\varepsilon^{\kappa}_{i,j,t} \sim \mathcal{N}(0, \sigma^{\kappa}_{\varepsilon})$.

Parameterizing the Course/Section Demand Model.

It remains is to specify a concrete distributional form for the error terms $\nu_{i,j,t}$ capturing heterogeneity in taste. We assume these distribute $\nu_{i,s,t} \sim TIEV$. The resulting conditional choice probabilities are of the standard logit form as considered below for a student who demands a section s under instructor j in academic term t.

$$\mathbb{P}(s \mid a_{i,t-1}, \mathbf{x}, t) = \frac{\exp\big(\Phi_{s,t} + \alpha_0 \cdot s_j(a_{i,t-1}, \mathbf{x}_{i,j,t}) + \alpha_1 \cdot f_j(a_{i,t-1}, \mathbf{x}_{i,j,t})\big)}{\sum_{s'} \exp\big(\Phi_{s',t} + \alpha_0 \cdot s_{j'}(a_{i,t-1}, \mathbf{x}_{i,j',t}) + \alpha_1 \cdot f_{j'}(a_{i,t-1}, \mathbf{x}_{i,j',t})\big)}.$$

7.2 Estimation via Maximum Likelihood

Our parametrization of the model and the distributional assumptions over the error terms suggest a simple estimation approach via Maximum Likelihood (ML). Under this approach we could proceed by maximizing the log of the likelihood associated to our observed data as specified in the following expression,

$$\mathcal{L}(\theta) = \sum_{i=1}^{N} \sum_{t=t_{i,0}}^{T_i} log\Bigg[\mathbb{P}\Bigg(j_t\ ,\ S^{\kappa_{i,t}}_{i,j_{i,t}} \Bigg|\ j_{i,\tau}, ..., j_{i,t_{i,0}}, a_{i,t_{i,0}}, \mathbf{x}_{i,j_{i,t},t}\ ;\ \theta \Bigg)\Bigg].$$

where θ denotes the vector of all parameters associated to each of the courses in the sequence $\mathcal{K}$ considered. In practice however, this approach faces some problems. First, considering the estimation of parameters for all courses simultaneously might pose numerical complications simply due to the number of these parameters. Second, notice that since we don't observe a student's type except for the initial ability $a_{i,t_{i,0}}$ as measured by the entrance exam record, a student's type increases across time. For example, a student's type in its second academic period can be thought as a pair $(a_{i,t_{i,0}}, \mathbf{x}_{i,j_{i,t_{i,0}}}, j_{i,t_{i,0}+1})$, describing the student's initial ability, the instructor the student is paired in its initial period $t-_{i,0}$, and the vector of covariates under which such a match takes place. Even for a modest number of courses in the sequence, the resulting computations required to code the gradient, as required for the implementation of an optimization routine, can be very cumbersome.

Faced with these considerations, we we opt for a sequential ML approach that involves iterating over the different courses in $\mathcal{K}$. At the κ-th iteration of the approach er estimate the primitives θ^{κ} associated to the κ-th course in the sequence. We then use these estimates, in particular those pertaining course κ's production functions, to update each student's ability as implied by the model. The latter estimates then serve as the basis for the $\kappa + 1$-th iteration of the algorithm where they are taken as observed data.

To formally illustrate this approach, let θ^κ denote the set of parameters associated with the κ-th course of the sequence $\mathcal{K}$. This includes both primitives of the course/section demand model and primitives of the learning production function model. We begin by constructing the log-likelihood based on the observed data for each student's first two consecutive academic periods upon enrolling in the university. For instance, if student i enrolls in course $\kappa = 1$ for the first time in academic period t, we utilize the data corresponding to academic terms t and $t+1$ for this student. Based on this, the data explained by the model for a student i with $t_{i,0} = t$ is given by: (i) $a_{i,t}$, the student's initial ability, (ii) $j_{i,t}$,$\mathbf{x}_{i,j_{i,t},t}$, the student's instructor match and vector of covariates for the first course enrollment instance, and (iii) $j_{i,t+1}$,$\mathbf{x}_{i,j_{i,t+1},t+1}$, the student's instructor match and vector of covariates for its second academic period. The average loglikelihood function for this data can be expressed in terms of the following,

$$\mathcal{L}(\theta^1, \theta^2) = \sum_{i=1}^{N} \sum_{t=1}^{2} \, log\Big[\mathbb{P}\big(j_{i,t} \mid a_{i,t-1}, \mathbf{x}_{i,j_{i,t},t} \, ; \, \theta^t\big) \cdot \mathbb{P}\big(S_{i,j_t}^{\kappa_{i,t}}, R_{i,j_t}^{\kappa_{i,t}} \mid j_{i,t}, a_{i,t}, \mathbf{x}; \theta^t\big)\Big].$$

where with a slight abuse of notation, we use $t \in \{0, 1\}$ to refer to the student i's first and second academic term as opposed to the actual academic period corresponding to these two enrollment instances. The first term inside the logarithm corresponds to the likelihood of observing the student's demand for the section they enroll in during academic term t. The second factor represents the likelihood of the observed course outcomes achieved by the student upon enrolling with a particular instructor in that term. Given that we only consider the first two academic terms of each student, our data contains observations solely for the first two courses in the sequence. Consequently, the log-likelihood function provided above depends exclusively on the parameters associated with these initial two courses: θ^1 and θ^2.

Implementing the sequential ML estimator for θ^1, θ^2 requires addressing a small identification concern. In essence, recall that in our identification results, disentangling the grading policy slopes (i.e., β_j) and the production function images (i.e., $f_j(a_{i,t}, \mathbf{x}_{i,j,t})$) for instructor in $\kappa = 2$ requires information on the student's performance on course $\kappa = 3$. However, the proposed sequential ML approach, the log-likelihood described above does not consider such information. This limitation arises because students can potentially reach course $\kappa = 3$ only in their third academic period, assuming they do not fail the first two courses in the sequence. This issue persists even if we were to estimate all parameters simultaneously, disregarding the sequential ML estimator, as it would still require the decoupling of grading policies and learning outcomes for the last course in the sequence considered.

To address this issue, we reparameterize the model for course $\kappa = 2$ in a way that renders an identified model while keeping θ^1 unchanged. Specifically, let's impose the restriction of $\beta_{j_2} = 1$ and c_{j^2} for all $\kappa = 2$ instructors. This accounts to defining the model for the second period in terms of production functions $\mathring{f}_{j^2}(a, \mathbf{x}) = \beta_{j^2} \cdot f_{j^2}(a, \mathbf{x}) + c_{j^2}$ that pool both learning output and grading policies into a single object. We denote the resulting vector of parameters for $\kappa = 2$ by $\mathring{\theta}^2$. Importantly, notice that the resulting model for student's first two academic terms can be used to identify the parameters θ^1 as our identification of the $\kappa = 1$ parameters doesn't require distinguishing $\kappa = 2$'s grading policies from production function images. We can thus optimize the log-likelihood function under the proposed parameterization and obtain consistent estimates for θ^1, $\hat{\theta}^1$.

Once the latter is achieved, we can transition to the second stage of the sequential ML approach where we estimate the parameters of the second course in the sequence. To accomplish this we use the estimates $\hat{\theta}^1$ for the learning production functions associated to any instructor j^1. These allow us to compute estimates for the implied learning outputs associated to each student's first academic period match. For example, after enrolling professor j^1's section for $\kappa = 1$, a student with an initial ability measurement of $a_{i,0}$ ends up with a new ability given by $a_{i,1} = f_{j^1}(a_{i,0}, \mathbf{x}_{i,j^1,1}\,; \hat{\theta}^1)$ where the notation makes it clear that we use the first step estimates in order to construct these ability estimates. At this point we can treat $a_{i,1}$, the ability estimates from the previous stage as observed data and use them to estimate θ^2 in the current stage of the algorithm. The process follows the same steps as before: (i) constructing the log-likelihood using data from two consecutive periods for all students upon their enrollment in a course $\kappa = 2$, and (ii) reparameterizing the model for θ^3 to account for the lack of identification of grading policies/ production functions for $\kappa = 3$.

7.3 Estimation Results

This section introduces the main findings resulting from the estimates of the model. The discussion is organized in terms of three main components: (i) estimates for the average learning production function, (ii) estimates for the distribution of learning outputs across professors, and (iii) estimates for the demand model primitives. Emphasis is placed on the implications of the estimates over the observed student-professor assignment and the possibility of improving such assignment via counterfactual policies explored in subsequent sections.

Average Learning Production Function Estimates

I start by documenting the predicted average learning production function where the average is taken across all Calculus 1 instructors. Figure 7 depicts the average learning production outcome as a function of a student's ability. Each panel conditions on a different value for the vector of covariates $\mathbf{x}$. For example, the second north-east panel corresponds to the subpopulation of female students majoring in STEM fields and who enroll in Calculus 1 under an instructor characterized by an above average teaching load, an above average general tenure, and a below average course-specific tenure. The x-axis of each panel displays a student's ability level. In turn the blue curve's height at any given ability level represents the average of the images for each instructor's learning production function according to the estimates (i.e., $\hat{f}_j(a, \mathbf{x})$). The shaded area depicts the associated 95% confidence interval.

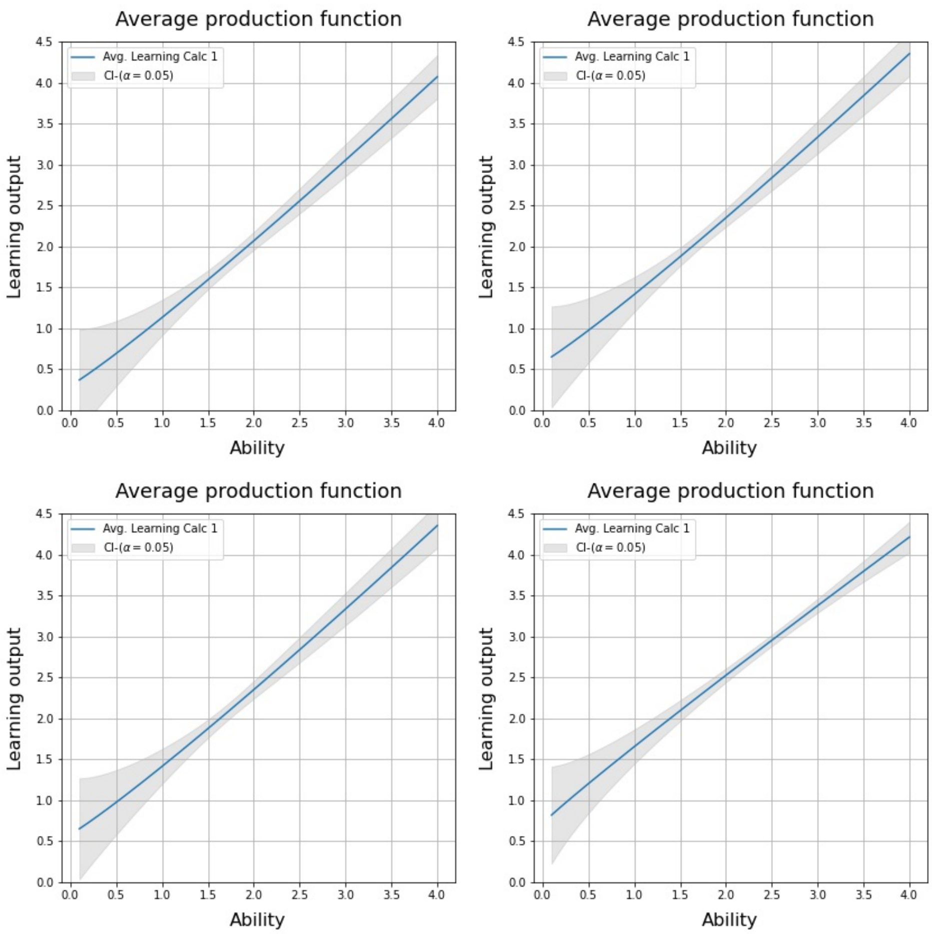


Figure 7: Average learning production function

Notes: The first (second) panel in the first row corresponds to the subpopulation of male (female) students majoring in STEM fields and who enroll in Calculus 1 under an instructor characterized by an above average teaching load, an above average general tenure, and a below average course-specific tenure. The first (second) panel in the second row corresponds to female engineering students under below average teaching-load instructors, below average general tenure, and below (above) average course-specific teaching tenure.

Two key observations emerge from these plots. First, each panel illustrates significant disparities in learning outcomes among students of varying ability levels. To put on numbers on this claim, consider in the first panel of the upper row a student of ability $a_i = 1.0$, which is at the lower end of the ability spectrum. When matched to a Calculus 1 as described by the panel, such a student can anticipate an average learning outcome of approximately 1.3 GPA points under the reference grading policy, or equivalently, a D grade. In contrast, a student of an ability of $a_i = 4.0$, positioned at the top of the distribution, achieves a learning outcome of approximately 3.8, equivalent to a B+ grade. More generally, the average learning production function is increasing relative to a student's ability level, and spans a wide range of distinct learning outputs. This is robust to changes in the vector of covariates as suggested by looking at the remaining panels.

Second, factors beyond a student's ability type influence the shape of the average production function. For instance, while the upper row panels exhibit slightly convex relationships, the bottom panels portray production functions under diminishing returns to a student's ability, as evident from the modest concavity of the average production function. In addition, the overall height of the production function varies across these panels. Consider for example a student with an ability of $a_i = 2.0$. In the north-west panel, this student achieves an approximate learning output of 2.0 GPA points under the reference professor, while in south-east panel, its average learning output increases to approximately 2.5.

The Distribution of Learning Outcomes Across Instructors

Gains from our reassignment counterfactual exercises depend on the existence of teaching ability differences across instructors. To understand these differences we need to look not at the average in the distribution of learning outcomes, but at the dispersion of the distribution. The left panel of Figure 8 shows this by plotting various percentiles in the distribution of learning outcomes. As before, the x-axis corresponds to the student's ability level. For any given ability, the images of the curves being depicted represent percentiles in the distribution of learning outputs for a student of such ability. While the Figure corresponds to a fixed vector of covariates, similar patterns arise when conditioning on a different vector of covariates.

Consistent with the stylized facts reported before, non-negligible variation exists in the learning outcomes a student can expect across different Calculus 1 instructors. For instance, a student with an ability level of $a_i = 2.0$ can expect an average learning outcome of approximately 2.2 GPA points when randomly paired with a Calculus 1 instructor. However, the range of possible learning outcomes extends from 2.0 GPA points at the 15-th percentile to 2.5 GPA points at the 85-th percentile. The latter difference is large, being equivalent to transitioning from a D to a C grade under the reference instructor's grading. The dispersion in learning outcomes becomes even more pronounced when considering students at the higher end of the ability distribution. For example, the same percentiles for a student with an ability of $a_i = 3.0$ correspond to a two-letter grade jump, ranging from 2.5 GPA points to 3.5 GPA points, or equivalently from C+ to B+ in terms of letter scores.

The second panel of Figure 8 complements the latter by displaying the distribution

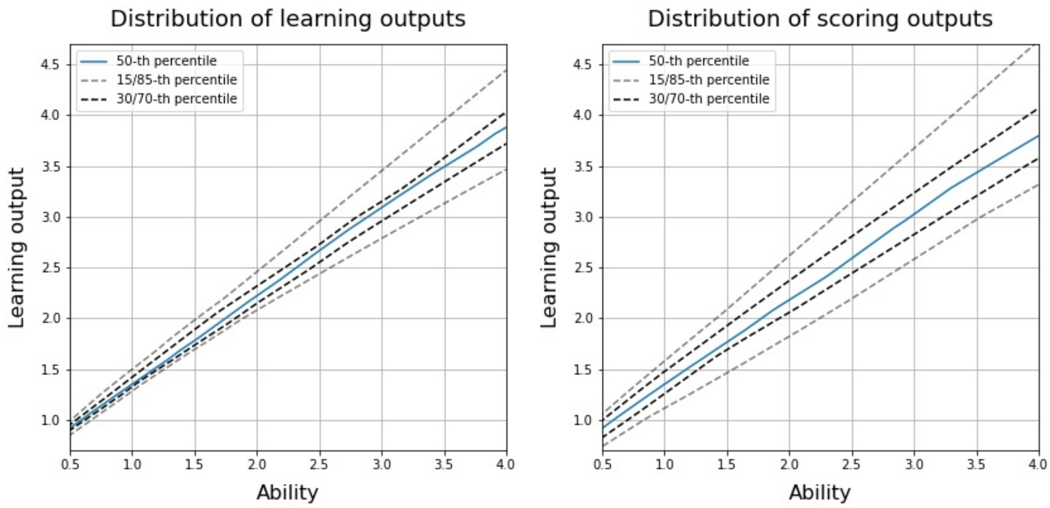


Figure 8: Distribution of learning outcomes

Notes: Both panels correspond to a female student in a STEM major, under above average teaching load, course tenure, and general tenure instructor.

of scoring outcomes that our randomly assigned student can anticipate. This provides an intuitive way of incorporating differences in instructors' grading policies into the presentation of our estimates. To start, notice that the average score output across instructors closely mirrors the average learning output curve in the first panel. This is consistent with the grading policy estimates, reported in the appendix, which suggest the average instructor's grading policy coincides with that of the reference professor. However, it's worth highlighting that the spread in the scoring distribution is higher than that of the distribution for learning outcomes. This holds true for all student ability levels and remains robust across changes in the conditioning vector of covariates, as illustrated in the appendix. For instance, for the student of ability $a_{i,0} = 2.0$ considered before, a move from the 15-th percentile to the 85-th percentile in the distribution of scores corresponds to a jump from approximately 1.6 to 2.6 GPA points.

Collectively, the estimates in Figure 8 indicate a potential source of tension for students concerning their preferences for both learning and scoring outcomes. In other words, if instructors associated with high-learning outcomes differ from those with high-scoring outcomes, students will encounter a trade-off between learning and scoring when selecting sections of Calculus 1. The ultimate choice a student makes will depend on the weight its preferences place on each of these aspects. Before delving into the preference side of this issue, let's analyze this trade-off by simply documenting the extent to which the optimal teaching and scoring instructors for a student differ, as well as the average learning/scoring magnitudes of these differences.

For instance, consider computing the score and learning outcomes for each course enrollment instance for Calculus 1 in our sample. We can then determine for each instance the number of instructors associated with a higher scoring output than that of the student's optimal professor in terms of learning output. Table 3 records this information. Each column in the table corresponds to a different integer value representing the number of professors who would improve a student's score, relative to the best instructor for the student in terms of the induced learning. The data in the k-th column for a given row is interpreted as the proportion of students within the subpopulation represented by the row who have k score-improving instructors relative to the learning optimal professor. To highlight variations in these score-improving opportunities across students of different abilities, the information is shown for different quartiles of the student ability distribution.

For the population of students as a whole, a large fraction of all Calculus 1 course-enrollment instances are such that the best instructors in terms of learning and scoring don't coincide. For instance, across all the course enrollment instances, 90.04% are associated to at least one score-improving instructor. Significant differences result from considering students of different ability levels, with the highest number of score-improving professors showing up among students at the top of the ability distribution. To some extent, the latter reflects the higher dispersion in the scoring and learning distributions according to the estimates.

Table 3: # of score-improving professors relative to the learning optimal professor.

	# of score-improving professors				
	1	2	3	4	≥5
All students	9.96	21.16	7.28	4.87	44.12
Ability Q1	16.94	21.40	10.49	6.70	26.26
Ability Q2	12.27	22.31	9.12	5.06	36.31
Ability Q3	6.25	21.79	8.39	6.88	49.81
Ability Q4	4.23	19.15	2.52	3.50	64.53

Notes: Information in the k-th column reports the fraction of students with exactly k score-improving professors relative to the learning optimal instructor.

It is also of value to think about the magnitude of the learning and scoring differences between the learning-optimal and scoring-optimal instructors. Intuitively, the larger the score gap corresponding to the deviation the higher is the temptation to deviate. Table 4 considers these two differences. The first column measures the difference in learning outcome between the learning-optimal and the scoring-optimal professor, which here is denoted as the scoring gap. The second column constructs the difference between the scoring outcomes of the scoring-optimal and the learning-optimal instructors, denoted as the scoring gap. Here, the results are revealing, as in all cases the scoring gap substantially exceeds the learning gap. To put this in context, let's entertain a Calculus 1 student of an average level who participates in the first-come-first-served mechanism. While opting for the best instructor in terms of learning involves a premium of approximately 0.24 GPA points in terms of learning, opting instead for the best scoring professor involves a premium of 1.12 GPA points in terms of scoring.

Table 4: Learning/Scoring gap between the learning and scoring optimal professors.

	Outcome gaps	
	Learning gap	Scoring gap
First term students	0.35	1.30
Ability Q1	0.21	0.98
Ability Q2	0.32	1.11
Ability Q3	0.37	1.34
Ability Q4	0.43	1.62
Non First-term students	0.24	1.12
Ability Q1	0.17	0.79
Ability Q2	0.23	1.06
Ability Q3	0.28	1.30
Ability Q4	0.36	1.62

Notes: The learning gap is defined as the average difference between the learning output a student obtains under the learning optimal instructor and the scoring optimal instructor for the academic term in which the course enrollment instance takes place. The scoring gap is the average difference between the score output a student obtains under the scoring optimal instructor and the learning optimal instructor for the academic period of the course enrollment instance considered.

The intuition above can also be framed in terms of the fraction of students who end sup being paired with the learning-optimal instructor in the data. For new students, this proportion is approximately 12.17%. Notably, this aligns with the fact that first-time students are assigned to sections within Calculus 1 randomly, and the average number of sections in the term when these students enroll in a course is 8.7. This corresponds to an 11.46% probability of being randomly matched with the learning-optimal instructor, very close to our estimate. Regarding students repeating the course, the fraction resulting from the estimates is approximately 16.20%. While a fraction above the case of randomly matched students, still very low.

Course/Section Demand Primitives

The key parameters of interest in our demand model are α_0, representing the marginal utility of expected scores, and α_1, indicating the marginal utility of expected learning outcomes. Table 5 reports the point estimates and standard errors for both parameters under Calculus 1 and Calculus 2 courses. For Calculus 1, both α_0 and α_1 exhibit positive values, signifying that students assign positive weight to both learning and scores when making course/section selections. Notably, the magnitude associated with α_0 is slightly above that of α_1 suggesting students place a higher weight on the students they expect to obtain above their pure learning outcomes.

Table 5: Estimation results - Course/section demand parameters

	Calculus 1	
Parameter	Estimate	Std. Error
α_0	1.23	0.05
α_1	1.20	0.10

I also highlight that these estimates indicate a substantial portion of the variation in demand decisions can be attributed to both the scoring and learning dimensions. For instance, when considering the other utility component, $\Phi_{s,t}$, the average value for these quantities among observed (s, t) pairs is -0.35, with a standard deviation of 1.78. These values are of the same order of magnitude to the score and learning outputs for most students in the sample.

8 Counterfactual - Dictatorial Assignment

Describing the Counterfactual Policy

Consider the problem of matching students and instructors under a dictatorial approach. The problem can be understood in terms of a university administration that seeks to pair students with instructors for a section of a course without considering students' preferences regarding their instructors. The administration's primary goal is to maximize the average student learning that results from the chosen assignment, as implied by the estimates of the learning production function. I am focusing on a myopic assignment problem in which matches are made on a period-by-period basis. This should be contrasted with the fully dynamic version where the assignment decision in one period considers its impact on the future pool of students seeking to enroll in the course (i.e., the set of students who fail or drop a course changes with different matches). The former approach significantly simplifies the computational challenges of the problem and can be understood as providing a lower bound for the improvements a planner can achieve through reassignments.

Specifically, let's consider the university's problem for the first course in the sequence, Calculus 1, during the academic period t. Denoting the pool of students seeking to enroll in a section of the course as $\mathcal{I}_t$, the university's objective is to select $\mu_{i,s} \in 0, 1$ for every $i \in \mathcal{I}_t$. In the latter, $\mu_{i,s} = 1$ is a dummy variable indicating that student i is assigned to section s according to the administration's match choice. When making these assignments, the university is bound by the exogenous capacity constraint for each section s, ensuring that it is not exceeded. To account for potential distributional concerns regarding the planner's objective, I examine various versions of the problem, differing in the weight, $\omega(a)$, assigned by the planner to each student's ability type. Denoting C_s as the capacity constraint for section s and j_s as the instructor associated with such a section, we can formulate the problem for period t as in the following,

$$\max_{\{\mu_{i,s}\}_{i,s}} \sum_{i \in \mathcal{I}} \sum_{s \in Sect_t} \omega(a_{i,t-1}) \cdot f_{j_s}(a_{i,t-1}, \mathbf{x}_{i,j_s,t}) \cdot \mu_{i,s},$$

subject to the constraints:

$$\sum_i \mu_{i,s} \leq C_s; \quad \forall\, s \in Sect_t,$$
$$\sum_s \mu_{i,s} \leq 1; \quad \forall\, i \in \mathcal{I}_t.$$

For the period t problem, the number of active sections (i.e., $Sect_t$) and the set of instructors leading these sections are exogenously given. In particular, in the simulation exercise, these coincide with the observed sections/instructor pairs for the corresponding academic period t. Given this, I can compute the learning outputs $f_{j_s}(a_{i,t-1}, \mathbf{x}_{i,j_s,t})$ for each potential student-professor match that could result from the planner's assignment choice. It is easy to see that this reduces the planner's problem to a simple assignment problem that can be solved using standard linear programming techniques[10].

After the planner chooses a specific assignment $\mu_{i,s_{i,s}}$, learning takes place according to the estimated production functions $\hat{f}_j$. In addition, students' score outcomes and retirement decisions are simulated by drawing from the estimated distributions for the scoring and dropping equation's error terms. These simulated outcomes determine the total number of students who need to re-enroll in the course in the subsequent term (i.e., due to either failing or dropping a course), which together with the first-time students observed in the data for the period $t+1$, determines $\mathcal{I}_{t+1}$. The planner then repeats the process by solving the assignment problem for students in period $t+1$ as described above.

Simulating the Counterfactual Policy

[10]The existence of an integer solution is guaranteed for assignment problems of this nature

Figure 9 shows the outputs resulting from the reassignment exercise. Three distinct reassignment simulations are considered, each characterized by a different weight function. In particular, I consider a weighting function $\omega(a) = \exp^{-\delta \cdot a}$, where $\delta \in \{0.0, 0.5, 2.0\}$. The first value in the set corresponds to a uniform weight, placing the same value for all student ability types in the reassignment exercise. In contrast, the latter two assign more weight to students with lower initial abilities, capturing a university's potential concern for improving learning outcomes for disadvantaged students. The first panel illustrates the average learning output for each student's ability level. While a dashed line represents the average under the observed assignment, the average under each counterfactual reassignment corresponds to a solid line. In addition, the second panel depicts the average learning difference between the counterfactual and the observed assignments, providing a clearer perspective on how various ability types experience gains or losses after implementing each policy.

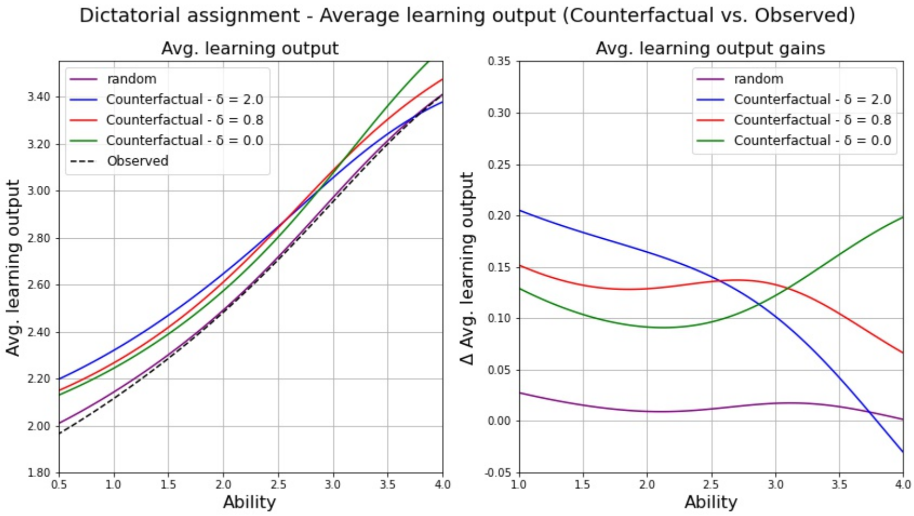


Figure 9: Dictatorial counterfactual - Avg. learning output

Two observations from the latter exercise can be highlighted. First, reassignments improve average learning outcomes, with the magnitude of these gains differing across students with varying initial abilities. For instance, consider the green curve corresponding to the uniform weight reassignment exercise. For students with ability levels described by $a_i = 4.0$, the gains correspond to an increase of more than 0.20 GPA points. Students with an ability level of 2.0 exhibit the smallest average gain, approximately 0.9 GPA points. Naturally, as I consider exercises that assign a higher weight to students with lower abilities, reassignment gains rise for low-ability students and decrease for high-ability students. For instance, under the extreme case of $\delta = 2.0$, which assigns very small weight to students with abilities exceeding 2.0, a student with an ability level of 4.0 witnesses no gains relative to the baseline, whereas the average learning return for a student with low ability, such as 1.0, increases to 0.20 GPA points.

Second, I emphasize that while learning gains vary across different student ability types, these gains remain positive across the entire spectrum of ability. At first, this may appear surprising, as it suggests that the university can improve average learning outcomes for all student types without facing the typical trade-offs associated with prioritizing the learning of one subgroup of students at the expense of another. However, the result becomes more intuitive when considering two key factors. First, under the observed assignment, a significant fraction of students enroll in Calculus 1 as first-time students. Consequently, they don't participate in the first-come-first-served mechanism and are instead randomly assigned to sections of Calculus 1. Random assignments are prone to generating many Pareto suboptimal pairings, allowing the planner to exchange slots between two students, i and i', to enhance the learning output of both. Second, even among reassignments involving students who are repeating Calculus 1 (i.e., approximately a third of the students in each academic term), the oversupply of instructors/slots, relative to the total number of students demanding sections, implies there is a significant amount of slackness in the capacity constraints of many sections under the observed assignment. This creates many opportunities in which the planner can modify a given student's assignment without taking a slot away from another student to meet the capacity constraint

In trying to understand the outcomes of the exercise, it is helpful to examine how the professor assigned to each student-ability type changes during the reassignment. To illustrate this, consider Figure 10, which shows the conditional ability distribution of students matched with each instructor under the uniform weight reassignment. In this figure, each row corresponds to a specific instructor, while the columns correspond to quartiles in the distribution of student ability. A cell associated with a given row/column is the proportion of all students matched with the instructor represented by the row who, upon enrollment, have an ability level belonging to the quartile specified by the column. I have ordered the instructors lexicographically, considering the percentage of students they enroll in each of the four quartiles (i.e., order first regarding the percent of students in Q1, Q2, etc.).

Notably, under the counterfactual reassignment, there is a discernible specialization in the professors assigned to students with different ability levels. This is evident from the diagonal structure observed in the heatmap of the first panel. This should be contrasted with the right-hand panel showing the conditional distributions under the observed assignment. In the latter, each instructor teaches a substantial number of students in each ability region. However, the distributions are not uniform, as the non-first-time students can choose the section they enroll in.

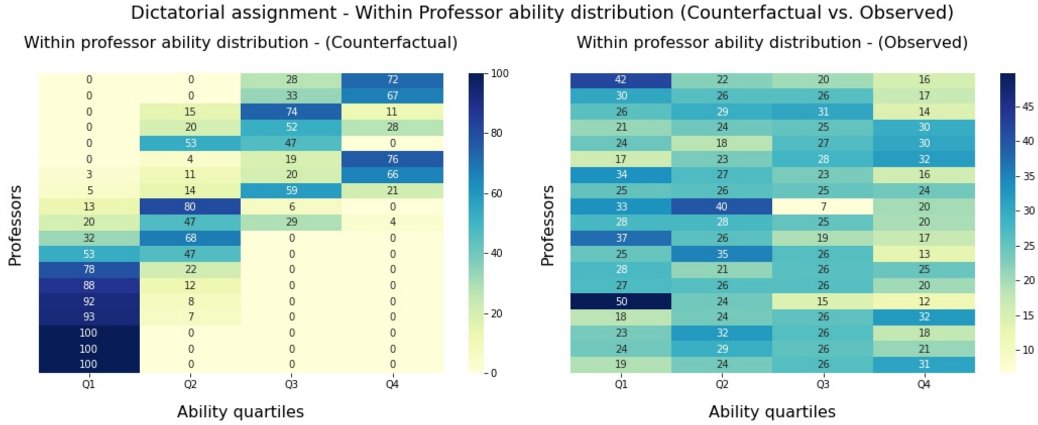


Figure 10: Dictatorial counterfactual - Within professor ability distribution

Although the reassignment is constructed with average learning outcomes as the objective, the gains are reflected in other related variables of interest to the university's administration. For instance, Figures 11 and 12 depict the changes in the rate at which students withdraw from sections of Calculus 1 and the average number of attempts required for the successful completion of Calculus 1, respectively.

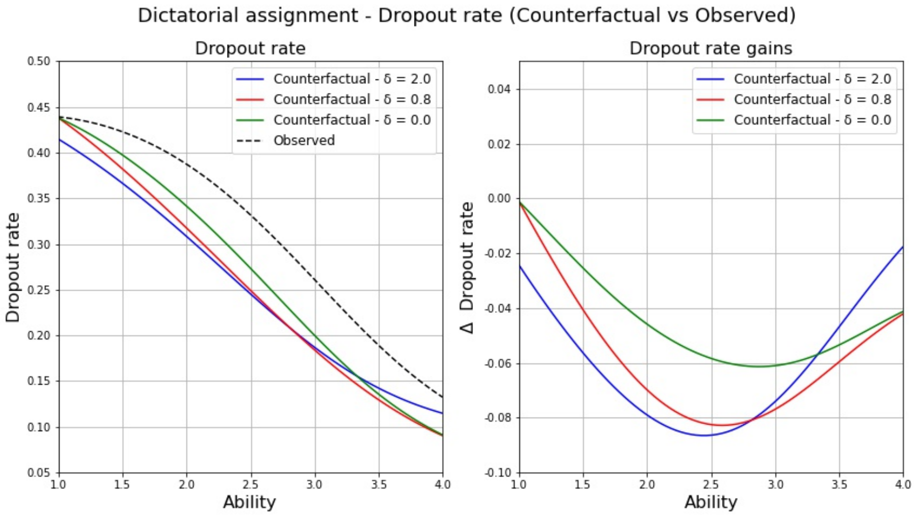


Figure 11: Dictatorial counterfactual - Course dropping rate

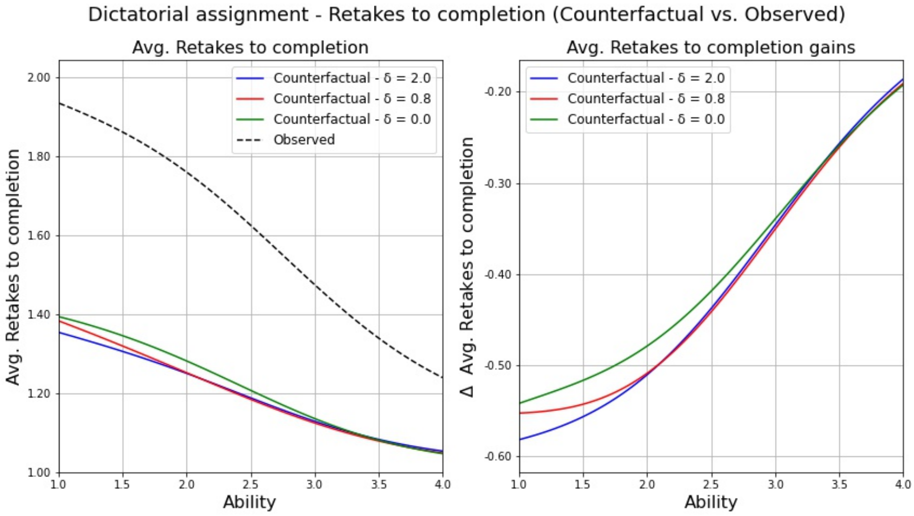


Figure 12: Dictatorial counterfactual - Attempts to completion

When examining the rate at which students opt to withdraw from their Calculus 1 sections, the reassignment of students yields significant improvements. This is particularly the case for students positioned within the mid-range of the ability distribution. As these are typically the students at the borderline between the course dropping and not dropping decisions, they are also more likely to change their dropping decision after experiencing the boost in their learning outcomes from the reassignment. For instance focusing on the uniform weight reassignment exercise, a student with an ability level of 2.0 drops the Calculus 1 section at a rate of 40% under the observed assignment assignment. This rate decreases to 34% under the uniform weight dictatorial reassignment.

Turning to the second plot, which illustrates the number of attempts a student requires to achieve a passing score in Calculus 1, we once again observe notable gains resulting from the counterfactual reassignment. For instance, let us revisit the example of a student with an ability level of 2.0, who under the baseline requires an average of approximately 1.75 attempts to successfully complete Calculus 1. Instead, under the counterfactual reassignment, the same student accomplishes this in 1.3 attempts. Substantial gains extend to various ability levels, as indicated by the right panel, with the smallest improvements observed among high-ability students who are less inclined to retake the course to begin with since they perform better. Nevertheless, it is worth noting that even for such students the reassignment leads to a significant drop in the number of attempts to completion. For example, a jump from an average of 1.21 retakes to slightly above 1.0 retakes for students of a top ability 4.0.

9 Conclusion

Higher-education institutions pair students with course professors using assignment rules with poorly understood learning properties. This is relevant because match effects in learning technologies imply that how students and professors are matched affects the university's learning outcomes. While other projects have explored the assignment problem in elementary and secondary education settings, structural differences relative to post-secondary settings hinder a simple extrapolation of these studies' conceptual and methodological insights. In this paper, I consider the measurement of matching effects in higher-education learning technologies and their use in evaluating the learning properties of assignment rules pairing students to course instructors. My empirical approach involves constructing a structural model that describes learning outcomes and course/section demand decisions under assumptions common to many post-secondary institutions.

Conceptually, I present arguments for identifying learning technologies in higher education where instructors, in addition to their teaching ability, vary in their grading policies. My proposal addresses the bias from using approaches based on within-professor score variation, common in elementary and secondary education projects but inadequate for post-secondary environments. In addition, I propose a novel channel through which grading policies can impact learning outcomes by influencing students' demand decisions in choice-based assignment mechanisms. Both insights inform my empirical work, demonstrating how, in my setting, learning outcomes can be improved through reassignments that fully consider the complementarities in the learning technology. While the empirical exercise occurs in a setting based on random and first-come-first-served rules, the framework and intuitions discussed are general and can be applied to studying the assignment problem in other post-secondary institutions.

Several unexplored research avenues appear particularly relevant. First, I have considered dictatorial policies reassigning students to instructors without using student choice. While informative about potential learning gains, the university might place value on mechanisms allowing students choice, making the proposal infeasible. One possible response consists of interpreting the results here as a benchmark or first best and exploring how commonly used choice-based assignment rules can replicate the benchmark assignment. For instance, I could simulate the learning outcomes of lottery or priority assignment rules used by multiple universities. Understanding how these mechanisms interact with student preferences could guide university assignment decisions without violating the constraint of granting students choice.

Second, although significant, learning efficiency is not the sole criterion for assessing the desirability of an assignment. For instance, universities might hold distributional and fairness in the access to high-quality instruction concerns when choosing across assignment rules, concerns which I have not considered directly. Additionally, extensive literature compares assignment rules based on student welfare and stability notions. An interesting research avenue involves combining these perspectives into a single framework for evaluating existing assignment rules.

Finally, in many settings, the classroom composition has a nontrivial influence over how instructors grade students. For example, an instructor might adjust grading in response to a section with students struggling with the material. Professors may also face explicit score distribution targets imposed by the university. The framework I propose can be adjusted to account for this. Furthermore, it is interesting to consider the implications of these relative grading policies for the policy gains I document.

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10 Appendix

10.1 Entrance Exam Scores Predict Course Scores

Table 6 illustrates the distribution of letter scores conditional on a student's initial ability measure. In this table, the columns correspond to the scores achieved by individual course-enrollment instances for students within the sample. The rows in the table delineate the segments within the distribution of initial ability where each student's entrance exam score falls.

Table 6: Calculus 1 letter score distribution conditional on ability

	Calculus 1			
	A	B	C	Fail/Retire
80% – 100%	46.49	19.78	17.77	15.96
60% – 80%	27.21	20.91	23.45	28.43
40% – 60%	21.42	18.37	23.79	36.43
20% – 40%	17.60	15.74	22.42	44.25
0% – 20%	11.74	13.36	23.12	51.77
	Calculus 2			
	A	B	C	Fail/Retire
80% – 100%	34.31	24.43	20.93	20.34
60% – 80%	18.90	22.25	26.99	31.86
40% – 60%	13.73	20.67	26.49	39.11
20% – 40%	11.30	18.36	26.92	43.42
0% – 20%	9.27	16.56	25.86	48.31

The table reveals a positive relationship between a student's obtained score and its initial ability. For instance, let's consider the case of Calculus 1. When we focus on students in the top 20.00% of the distribution of initial abilities, they exhibit a 46.00% likelihood of attaining a score of A. In contrast, this probability decreases significantly to 11.74% when considering students at the lower end of the ability distribution. Notably, roughly 75.00% of students in the lower segment of the ability distribution ultimately receive either a C grade or a score associated with course failure or withdrawal. This pattern is mirrored in the context of Calculus 2. Here, students positioned at the top and bottom of the ability distribution exhibit odds of 34.31% and 9.27%, respectively, for obtaining a score of A.

10.2 Identification Under Course Dropping

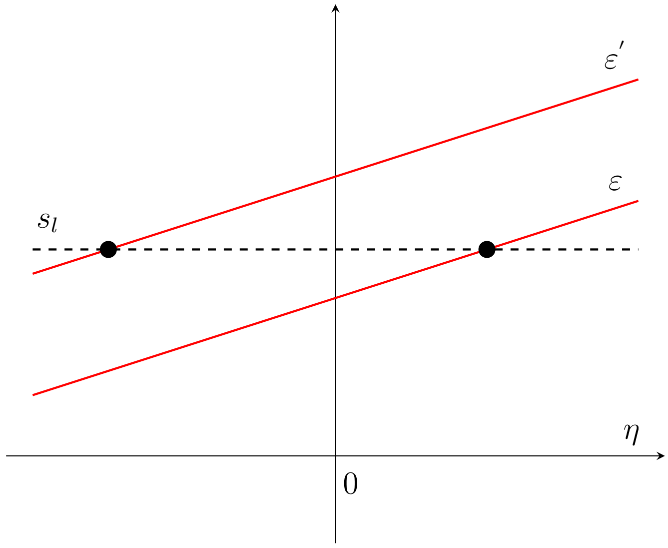
Thus far we have only provided arguments for the identification of our empirical model in a setting where students are unable to drop sections previously enrolled. In situations in which a non-negligible number of students choose to drop a section, ignoring this might be problematic. Intuitively, the issue arises from the fact that all our identification results are based on our observations of the group of students who achieve scores above a certain threshold, denoted as s_l. Yet, when students have the option to drop a course, the researcher can only observe the fraction of students who score above s_l conditional upon not dropping the course.

Failure to account for this distinction can lead to upward bias in our estimators, where estimates suggest that the learning returns of professors are higher than they truly are. Moreover, depending on the relationship between grading policies and professor productivity, this bias can result in erroneous conclusions regarding disparities in learning returns across instructors. For instance, if students are more likely to drop a course under the instruction of subpar professors, we run the risk of underestimating the gap in instructional quality between a high return and a low return instructor.

We now try to extend the arguments in the previous subsection in a way that accommodates for the truncation issue resulting when students can drop a course. Following the same arguments used before, consider the mass of students of initial type $a_{i,0} = a_0$ who in their first academic term obtain a score of at least s_l after enrolling instructor j^1's section under the covariate vector $\mathbf{x}_1$. Following the discussion above, we also condition on the subgroup of students who choose not to drop j^1's section as otherwise we wouldn't observe a score record for the student. Under our model, the latter conditional probability can be written as follows,

$$\begin{aligned}
&\mathbb{P}\big(S^1_{i,j^1} \geq s_l \mid a_0,\ \mathbf{x}_1,\ j^1,\ R^1_{i,j^1} = 0\big) \\
&= \int_{\varepsilon^1} \int_{\eta^1} \mathbf{1}\big\{R^1_{i,j^1} = 0\big\} \mathbf{1}\big\{S_{i,j^1} \geq s_l\big\}\ f_\eta(\eta^1) f_\varepsilon(\varepsilon^1)\, d\eta^1\, d\varepsilon^1 \\
&= \int_{\varepsilon^1} \mathbf{1}\left\{\varepsilon^1 \geq \frac{s_{l^*} - \beta_{j^1} f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\varepsilon}\right\} \left[1 - F_\eta\left[\frac{s_l - \beta_{j^1} \cdot f_{j^1}(a^0, \mathbf{x}_1) - c_{j^1} - \sigma^1_\varepsilon \cdot \varepsilon^1}{\sigma^1_\eta}\right]\right] f_\varepsilon(\varepsilon^1) d\varepsilon^1.
\end{aligned} \tag{1}$$

This expression closely resembles the one derived in our previous identification when trying to identify the marginal student for score s_l. It however differs in two crucial aspects that complicate this interpretation. First, students (within the conditioning set) now differ in two unobserved ways: their η^1 and ε^1 draws. Given that conditional on not dropping the course, ε^1 has an impact on the final score received by a student, this means we now should think not of a single marginal student for score s_l but of a marginal student for s_l under each potential draw of ε^1. Our observation of the mass of student scoring above s_l now corresponds to adding up the mass of students who score above s_l across all this different sub populations based on ε^1. Figure ..., the analog to Figure ... in the main subsection illustrates this by showing how a change in ε^1 shifts the linear function defining the marginal student. Second, the integral on the right hand side must now reflect the fact that we only consider students who choose not to drop the course. This is captured by the indicator term inside the integral $\mathbf{1}\{R^1_{i,j^1} = 0\}$. In terms of the graph below, this amounts to only counting the students who score above s_l for some of the ε^1 sub populations, namely, those who choose not to drop the course.



In practical terms, the implication is that we cannot directly invert the previous equation to learn about σ^1_η and $f_{\hat{j}^1}(a_0, \mathbf{x}_1)$ as considered in our previous arguments. Some additional work is required in order to accomplish this. Nevertheless, what we can do is to identify the marginal student, not in terms of obtaining a score s_l, but in terms of choosing to drop the course. To see this, consider an expression for the mass of students who choose not to drop j^1's section under our conditioning set,

$$\begin{aligned}\mathbb{P}\big(R^1_{i,j^1} = 0, \mid a_0,\ \mathbf{x}_1,\ j^1\big) &= \int_\varepsilon \mathbf{1}\left\{\varepsilon \geq \frac{s_{l^*} - \beta_{j^1} \cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\varepsilon}\right\} f_\varepsilon(\varepsilon) d\varepsilon \\ &= 1 - F_\varepsilon\left[\frac{s_{l^*} - \beta_{j^1} \cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\varepsilon}\right]\end{aligned}$$

Inverting the expression above delivers an expression for the ε^1 corresponding to the student who just marginally chooses not to drop j^1's section of course $\kappa = 1$.

$$\frac{s_{l^*} - \beta_{j^1} \cdot f_{j_1}(a_0, \mathbf{x}_1) - c_{j_1}}{\sigma^1_\varepsilon} = F^{-1}_\varepsilon\Big[1 - \mathbb{P}\big(R^1_{i,j^1} = 1, \mid a_0,\ \mathbf{x}_1,\ j^1\big)\Big]. \tag{2}$$

We can now make some progress by combining the identified expressions just derived. In particular we can use the latter to pin down the identity of the average marginal student relative to obtaining a score weakly above s_l. Once this is achieved, the same steps followed in the previous section can be used to infer the the variance parameters and the production function of at least one $\kappa = 1$ professor, namely the reference professor $\hat{j}_1$. Proposition 5 formally states and proves this claim.

Proposition 5. *The image $f_{\hat{j}_1}(a_0, \boldsymbol{x}_1)$ and the variance parameters $\sigma^1_\eta, \sigma^1_\varepsilon$ are point identified.*

Proof. Consider equation 1 describing the fraction of students (within the conditioning set) that obtains a score weakly above s_l in j^1's $\kappa = 1$ section. In particular, we consider the case for $l = l^*$, the cutoff above which students obtain a pass score. By algebraically manipulating this expression we obtain what follows,

$$\mathbb{P}\big(S^1_{i,j_1} \geq s_{l^*} \mid a^0,\ \mathbf{x}_1,\ j^1,\ R^1_{i,j^1} = 0\big)$$
$$= \int_{\varepsilon^1} \mathbf{1}\Bigg\{\varepsilon^1 \geq \underbrace{\frac{s_{l^*} - \beta_{j^1} \cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\varepsilon}}_{I}\Bigg\}\Bigg[1 - F_\eta\Bigg(\frac{\sigma^1_\varepsilon}{\sigma^1_\eta} \cdot \Bigg\{\underbrace{\frac{s_{l^*} - \beta_j \cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\varepsilon}}_{II} - \varepsilon^1\Bigg\}\Bigg)\Bigg]\, f_\varepsilon(\varepsilon^1) d\varepsilon^1.$$

Notice that terms I and II (both of which coincide) are quantities we have previously identified in equation 2. We can treat them as known quantities in the equation above. Since the left hand side is also an observed quantity (i.e., crucially, this is true because the researcher is capable of observing scores for students who don't drop the course), we can treat the identity above as just a function of the quotient $\sigma^1_\varepsilon / \sigma^1_\eta$. Furthermore, it is easy to see that under the region of integration considered, term II is always below ε^1 which implies that an increasing of the quotient $\sigma^1_\varepsilon / \sigma^1_\eta$ corresponds to a pointwise decrease of the integrand considered in the right hand side. It follows from standard inversion arguments that we can use the identity above to identify the true value of the quotient $\sigma^1_\varepsilon / \sigma^1_\eta$.

Let's now consider the analog to the previous expression for an arbitrary scores threshold s_l. This is given by the equation below,

$$\mathbb{P}\big(S^1_{i,j^1} \geq s_l \mid a_0,\ \mathbf{x}_1,\ j^1,\ R^1_{i,j^1} = 0\big),$$
$$= \int_{\varepsilon^1} \mathbf{1}\Bigg\{\varepsilon^1 \geq \frac{s_{l^*} - \beta_{j^1} \cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\varepsilon}\Bigg\}\Bigg[1 - F_\eta\Bigg(\frac{\sigma^1_\varepsilon}{\sigma^1_\eta} \cdot \Bigg\{\underbrace{\frac{s_l - \beta_{j^1} \cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\varepsilon}}_{\theta(s_l \mid a_0, \mathbf{x}_1, j^1)} - \varepsilon^1\Bigg\}\Bigg)\Bigg]\, f_\varepsilon(\varepsilon^1) d\varepsilon^1.$$

It is clear from the preceding discussion that both terms $(s_{l^*} - \beta_j f_{j_1}(a^0, \mathbf{x}) - c_j)/\sigma^1_\varepsilon$ and $\sigma^1_\varepsilon / \sigma^1_\eta$ inside the integral term can be treated as known quantities. The key observation is then that we can treat the right hand side as a monotone function of the quotient $(s_l - \beta_j f_{j_1}(a^0, \mathbf{x}) - c_j)/\sigma^1_\varepsilon$ and $\sigma^1_\varepsilon / \sigma^1_\eta$ for any score cutoff s_l we entertain. We can then follow the same arguments as in section **??** by considering the system of equations defined by,

$$\theta(s_l|a_0,\mathbf{x}_1,j^1)=\frac{s_l-\beta_{j^1}f_{j^1}(a_0,\mathbf{x}_1)-c_{j^1}}{\sigma^1_\eta} \text{ and } \theta(s_{l'}|a_0,\mathbf{x}_1,j^1)=\frac{s_{l'}-\beta_{j^1}f_{j^1}(a_0,\mathbf{x}_1)-c_{j^1}}{\sigma^1_\eta}.$$

As before, it is easy to see that by considering $s_l \neq s_{l'}$, the system above has a unique solution in terms of quantities σ^1_ε and $\beta_{j^1}\cdot f_{j^1}(a_0,\mathbf{x}_1)+c_{j^1}$. The former, together with our previous identification of the quotient $\sigma^1_\varepsilon/\sigma^1_\eta$, allows us to recover the variance term σ^1_η. In turn the result for $\kappa=1$'s reference professor follows from our grading policy normalization of $(\beta_{\hat{j}^1},c_{\hat{j}^1})=(1,0)$ ■

We can mimic the marginal logic student logic followed in Section **??** when trying to understand the content of Proposition 5. In doing so, it is useful to recall the main challenges arising from the possibility of student's dropping a course: (i) we don't observe the score of students who drop the course, (ii) the unobserved draw affects a student's incentive to drop the course. The first part of Proposition 5 shows we can easily correct for the first issue by using our observations of who students are dropping. In other words, we can infer who the marginal student dropping the course is and use this observation when constructing an identity describing the marginal student obtaining a score of s_l. The second part shows that even when ε draws affect the scoring equation, for our purposes we can focus in identifying the marginal student for a ε draw of zero.

The remainder of the argument tracks closely our previous work on Section **??** in that we used data for the student's performance on the second course of the sequence to disentangle $\kappa=1$'s grading policies and production functions. For instance, consider all students in the conditioning set who after obtaining a pass score for $\kappa=1$, enroll j^2's of $\kappa=2$ under covariates $\mathbf{x}_2$. We are interested in an expression for the fraction of these students who obtain a score weakly above s_l. Our model implies the following expression,

$$\begin{aligned}\mathbb{P}\big(S^2_{i,j^1}\geq s_l \mid a_0,\ \mathbf{x}_1,\ \mathbf{x}_2,\ j^1,\ j^2,\ R^2_{i,j^2}=0\ S^1_{i,j^1}\geq s_{l^*}\big)\\ =\int_{\varepsilon^2}\mathbf{1}\left\{\varepsilon^2\geq\frac{s_{l^*}-\beta_{j^2}\cdot f_{j^2}(f_{j^1}(a_0,\mathbf{x}_1),\mathbf{x}_2)-c_{j^2}}{\sigma^2_\varepsilon}\right\}\\ \times\left[1-F_\eta\left(\frac{s_l-\beta_{j^2}\cdot f_{j^2}(f_{j^1}(a_0,\mathbf{x}_1),\mathbf{x}_2)-c_{j^2}-\sigma_\varepsilon\cdot\varepsilon^2}{\sigma^1_\eta}\right)\right]f_\varepsilon(\varepsilon^2)d\varepsilon^2.\end{aligned}$$

Exactly the same arguments as in the preceding discussion can be applied to the $\kappa=2$ problem. While direct inversion of the expression above is not possible, we can infer the primitives of interest by using our observations for how many students choose to drop j^2's section. After achieving this the results are just as those considered in Section 6 in that we can identify $\kappa=1$ production functions given injectivity of the $\kappa=2$ instructor's production function. Below we state the result without a proof as it is identical to the arguments already outlined.

Proposition 6. *The following identification results hold,*

1. *The image of the composition* $\beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0, \boldsymbol{x}_1), \boldsymbol{x}_2) + c_{j^2}$ *and the variance term* $\sigma_\eta^2, \sigma_\varepsilon^2$ *are point identified,*

3. *Suppose that the learning production function* $f_{j^2}(\cdot, \boldsymbol{x}_2)$ *is injective. Then the image* $f_{j^1}(a_0, \boldsymbol{x}_1)$ *is point identified provided the existence of* $\tilde{a}_0$ *such that* $f_{j^2}(f_{j^1}(a_0, \boldsymbol{x}_1), \boldsymbol{x}_2) = f_{j^2}(f_{\hat{j}^1}(\tilde{a}_0, \boldsymbol{x}_1), \boldsymbol{x}_2)$.